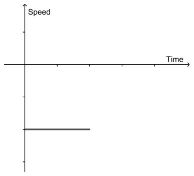
AP Quiz 3: Mathematical/Intuitive Kinematics

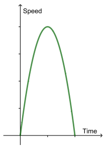
Your name-period here:

1 *Throwing up*

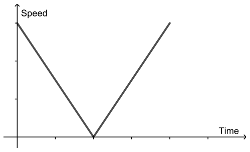
1. *(3pt) Which of these correctly represents the graph of speed vs time for an object that is thrown straight upwards and returns to Earth? Neglect air resistance. All graphs end immediately before the object hits the ground.*



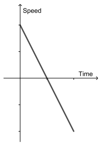
(a)



(b)



(c)



(d)

$\boxed{\text{C}}$ The first graph is acceleration for a falling object. The second is position. The fourth is velocity. Remember that speed is instantaneous velocity without direction, so it is always positive.

2 Woolley's nightmare

1. *(4pt) A fighter pilot is flying at a constant velocity of* $300\ \frac{\text{m}}{\text{s}}\hat{\text{x}}$ *when an unseen aircraft* 500 m *behind the first, and traveling at the same velocity, fires a short-range missile at the first aircraft. The missile's acceleration is* $10\ \frac{\text{m}}{\text{s}^2}\hat{\text{x}}$ *and the missile's maximum range when fired from a stationary platform is* 1000 m. *Does the missile hit the aircraft before running out of fuel? If so, at what time? Ignore air resistance and show all work.*

First we move to the reference frame of the aircraft that is about to fire the missile to eliminate the initial velocities. Since the missile will start with the same initial velocity as the plane, that velocity disappears as well. In that reference frame:

$$\vec{r}_p = \vec{r}_{p,0}$$

$$\vec{r}_m = \frac{1}{2}\vec{a}_m t^2$$

This immediately tells us how far the missile must travel. It must cover the distance between the two planes, which is 500 m.

$\boxed{\text{Yes}}$ The missile hits the plane.

Now we find the time by setting the position vectors equal.

$$\vec{r}_p = \vec{r}_m$$

$$\vec{r}_{p,0} = \frac{1}{2}\vec{a}_m t^2$$

$$500\ \text{m} = \frac{1}{2}10\ \frac{\text{m}}{\text{s}^2}\text{t}^2$$

$$\boxed{t = 10\ \text{s}}$$

3 Of mice and physics

1. (3pt) *Two mice are racing for the same cheese. In the reference frame of the first mouse, the cheese is* $\Delta x_{1,0}\hat{\mathrm{x}}$ *away. The first mouse moves with a velocity of* $v_{1,0}\hat{\mathrm{x}}$*. The second mouse starts from rest and moves with a constant acceleration* $a_2\hat{x}$ *and arrives after the first mouse by a time of* Δt_{12}*. The time for the first mouse to arrive was* t_1*. How far did the second mouse start from the cheese?*

   A. $\frac{1}{2}a_2\left(\Delta t_{21}\right)^2$

   B. $\frac{1}{2}a_2\left(\frac{\Delta x_{1,0}}{v_{10}} - \Delta t_{21}\right)^2$

   $\boxed{\text{C.}}$ $\frac{1}{2}a_2\left(\frac{\Delta x_{1,0}}{v_{10}} + \Delta t_{21}\right)^2$

   D. $\frac{1}{2}a_2\left(\frac{\Delta x_{1,0}}{v_{10}}\right)^2$

4 Planes in the day

Two aircraft are flying over a city. One of them has an initial position vector of

$$\vec{r}_{1,0} = 5\ \mathrm{m}\hat{\mathrm{x}} - 15\ \mathrm{m}\hat{\mathrm{y}} + 78.5\ \mathrm{m}\hat{\mathrm{z}}$$

and a velocity of

$$\vec{v}_1 = 4\ \frac{\mathrm{m}}{\mathrm{s}}\hat{\mathrm{x}} + 2\ \frac{\mathrm{m}}{\mathrm{s}}\hat{\mathrm{y}}$$

The other has a position vector of

$$\vec{r}_{2,0} = 5\ \mathrm{m}\hat{\mathrm{x}} + 5\ \mathrm{m}\hat{\mathrm{y}} + 28.5\ \mathrm{m}\hat{\mathrm{z}}$$

and a velocity of

$$\vec{v}_2 = 4\ \frac{\mathrm{m}}{\mathrm{s}}\hat{\mathrm{x}} - 2\ \frac{\mathrm{m}}{\mathrm{s}}\hat{\mathrm{y}} + 4\frac{\mathrm{m}}{\mathrm{s}^2}\mathrm{t}\hat{\mathrm{z}}$$

1. *(6pt) Do they collide? If so, at what time? Show all work!*

   Once again we need to change reference frames. Let's change to the reference frame of the first object. To do this we subtract the initial position vector of the first object from the second and the velocity of the first object from the second

$$\vec{r}_{210} = \vec{r}_{20} - \vec{r}_{10}$$

$$\vec{v}_{21} = \vec{v}_2 - \vec{v}_1$$

$$\vec{r}_{210} = 0\ \hat{\mathrm{x}} + 20\ \mathrm{m}\hat{\mathrm{y}} - 50\ \mathrm{m}\hat{\mathrm{z}}$$

$$\vec{v}_{21} = 0\ \hat{\mathrm{x}} - 4\ \frac{\mathrm{m}}{\mathrm{s}}\hat{\mathrm{y}} + 4\ \frac{\mathrm{m}}{\mathrm{s}^2}t\hat{\mathrm{z}}$$

   So the whole position vector of the second aircraft in the frame of the first is

$$\vec{r}_{21} = \left(20\ \mathrm{m} - 4\ \frac{\mathrm{m}}{\mathrm{s}}t\right)\hat{\mathrm{y}} + \left(-50\ \mathrm{m} + 2\ \frac{\mathrm{m}}{\mathrm{s}^2}t^2\right)\hat{\mathrm{z}}$$

In the object's reference frame, collision happens when the second object's position is exactly 0. In our case that immediately gives two equations (dropping units)

$$20 = 4t_y$$

$$50 = 2t_x^2$$

$$t_y = 5 \text{ s}$$

$$t_x = 5 \text{ s}$$

Yes, at

$$\boxed{t = 5 \text{ s}}$$

5 Throwing a ball in the wind part 2

1. *(4pt) You throw a ball with a mass of M horizontally with a velocity $\vec{v} = v_x \hat{\text{x}}$ across a vertically oriented wind tunnel on Earth where the gravity is $-g\hat{\text{z}}$. The wind in the tunnel results in a z force of $F_z\hat{\text{z}}$ which you can take as constant. The width of the wind tunnel is Δx. What will be the displacement vector from the point where the ball was thrown to the point where it hits the wall?*

   The displacement vector will be given by

$$\Delta\vec{r} = v_x t\hat{\text{x}} + v_y t\hat{\text{y}} + \frac{1}{2}t^2\left(\frac{F_z}{M} - g\right)\hat{\text{z}}$$

We can solve for t using that the x component of the displacement must be Δx when the ball hits the wall

$$\Delta x = v_x t$$

$$t = \frac{\Delta x}{v_x}$$

$$\boxed{\Delta\vec{r} = \Delta x\hat{\text{x}} + \Delta x\frac{v_y}{v_x}\hat{\text{y}} + \frac{1}{2}\left(\frac{\Delta x}{v_x}\right)^2\left(\frac{F_z}{M} - g\right)\hat{\text{z}}}$$

(0pt) Calculate the average thickness of **your name and period, written at the top of your paper.** Note for some of you: I do not teach 3rd or 5th period.