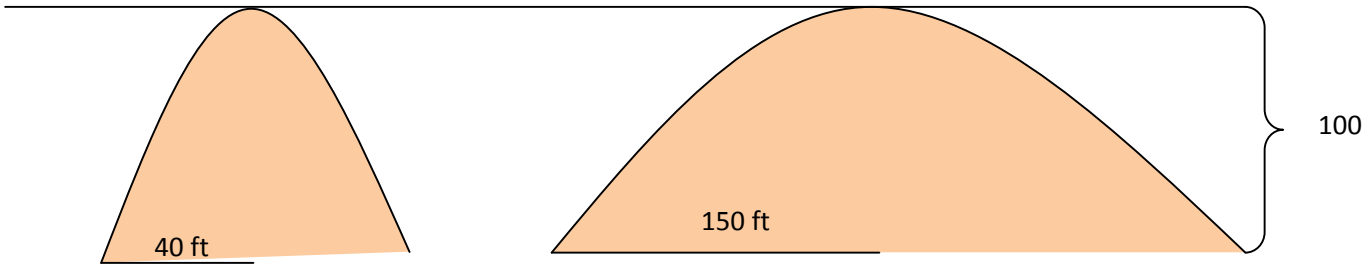
Slopes of Lines

Suppose you and your friend are riding a bicycle up a hill. You are faster than your friend, so you get to the other side of the hill far before he even reaches the hill. He calls you on your cell phone and asks you how steep the hill is. You respond, "100 feet hight." What is wrong with your answer?



Both of the hills above are 100 feet high, but one is steeper than the other. Which one would you consider steeper?

If you answered the one on the left, ask yourself why you think this is so.

On the first hill, you only travel 40 feet horizontally, but your elevation changes by 100 feet!

On the second hill, you travel 150 feet horizontally before you reach the top at 100 feet.

Therefore, in order to express steepness of a hill, you need to discuss the change in horizontal distance AND the change in height.

The slope of the first hill is 100 feet over 40 feet, or

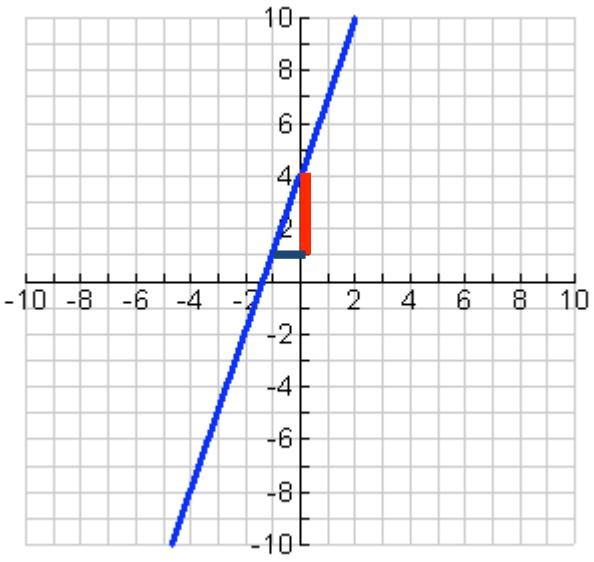
$$\frac{100}{40} = \frac{5}{2} = 2.5$$

The slope of the second hill is 100 feet over 150 feet, or

$$\frac{100}{150} = \frac{2}{3} \simeq .667$$

The slope is, therefore, the ratio of vertical change over horizontal change.

How do we relate this to lines on the plane?



If we want to find the slope of this line, we need to find the ratio of the vertical change over the horizontal change. First I need to pick two points.

Note that the points that are clearly on the line are (-1, 1), (0, 4), (1, 7).

Find the ratio of the vertical change over the horizontal change for (-1,1) and (0,4).

The vertical change is 3 units, and is derived by *4 -1 = 3*

The horizontal change is 1 unit, and is derived by *0 - -1 = 1.*

Therefore, the slope is

$$\frac{4 ! 1}{0 ! ! 1} = 3$$

If we pick two other points, (-1,1) and (1,7) we get

$$\frac{7-1}{1--1} = \frac{6}{2} = 3$$

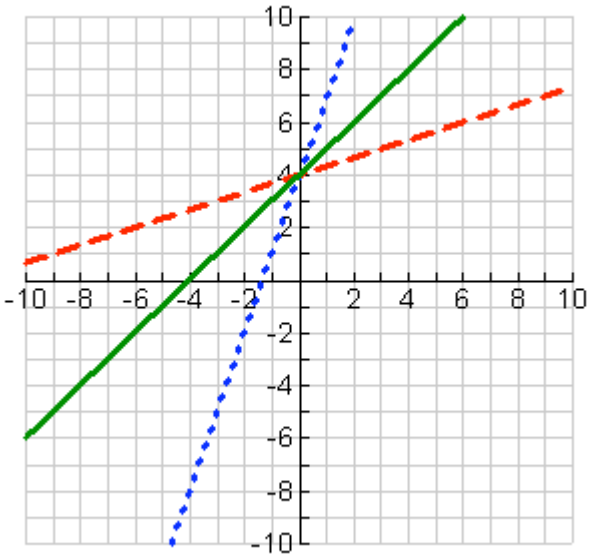
Regardless of which two points I choose, we always get the same slope.

(0,4) , (1,7) will give slope

$$\frac{7-4}{1-0} = 3$$

Why is this?

If we draw the right triangles generated by these points, drawn like the one above, we will see that they are all similar. If two polygons are similar, the ratios of their corresponding sides are equal. Since the slope is the ratio of the length of the legs of the right triangles, they will always be the same. Therefore, the slope is independent of your choice of points.



Find the slope of each line.

Solid line:

Dashed line:

Dotted line:

What is the relationship between their slopes?

Is it expected.

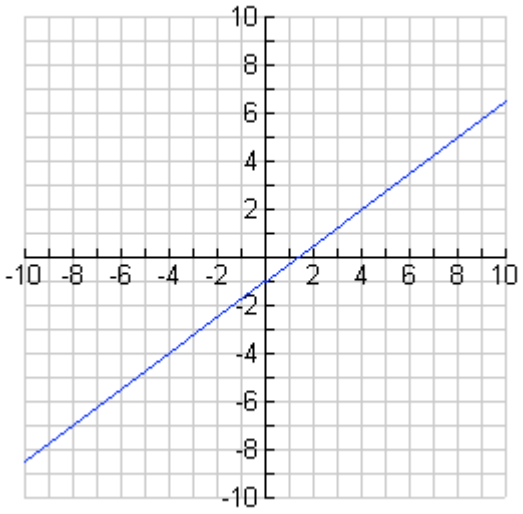
In general, if (x_1, y_1) and (x_2, y_2) are on a line, then the formula for the slope, *m*, is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Practice:

1) Find the slope of each line :

a)



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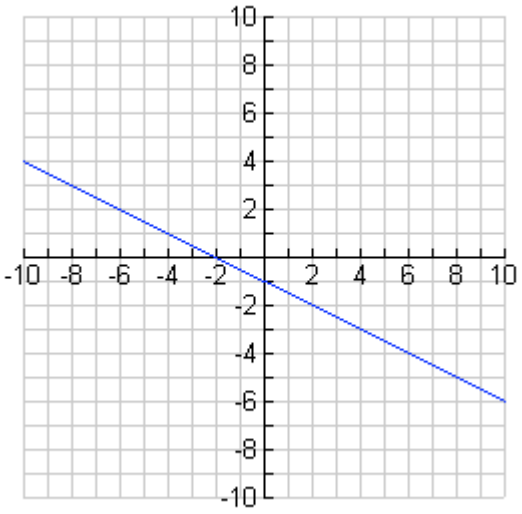
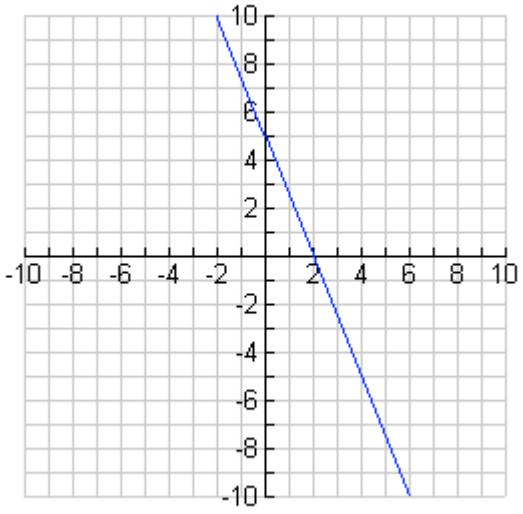
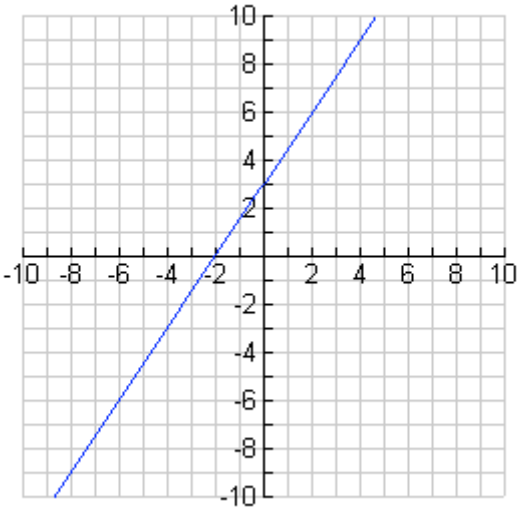


c)



d)



2) Find the slope of the line through these points

a) (2,-3) and (1,5) b) (-1,4) and (-5,-2) c) (- ½ , 2) and (¾ , -5) d) (.53, 6) and (1.37, 6.7)