BACHELOR OF COMPUTER APPLICATIONS (BCA)

CSB-1113 (Mathematics - I)

Block 1: Introduction to Sets Functions and Numbers

Unit 2: Principle of Mathematics Induction

Structure

2.1 INTRODUCTION

The Principle of Mathematical Induction is a powerful and widely-used technique in mathematics, particularly in the study of sequences, series, and discrete structures. It provides a systematic method for proving statements that hold for infinitely many cases, typically involving natural numbers.

The principle is based on a simple yet profound idea: if we can show that a statement holds for a base case, and then demonstrate that if the statement holds for any particular case, it also holds for the next case, then we can conclude that the statement holds for all cases beyond the base case.

2.2 OBJECTIVES

After going through this unit, students will be able to;

- Understand the Principle of mathematical induction
- Apply the Principle of mathematical induction to solve various numerical problems

2.3 PRINCIPLE OF MATHEMATICAL INDUCTION

Let $P(n)$ be a statement involving the natural number n such that

(i) $P(1)$ is true and

(ii) $P(k+1)$ is true, whenever $P(k)$ is true

then $P(n)$ is true for all $n \in N$.

Example 1: Using the principle of mathematical induction, prove that

$$1 + 3 + 5 + 7 + \cdots + (2n - 1) = n^2 \text{ for all } n \in N.$$

Solution: Let the given statement be $P(n)$. Then,

$$P(n): 1 + 3 + 5 + 7 + \cdots + (2n - 1) = n^2.$$

Putting $n = 1$ in the given statement, we get

$$\text{LHS} = 1 \text{ and RHS } = 1^2 = 1.$$
$$\therefore \quad \text{LHS} = \text{RHS}.$$

Thus, $P(1)$ is true.
Let $P(k)$ be true. Then,

$$P(k): 1 + 3 + 5 + 7 + \cdots + (2k - 1) = k^2. \qquad (1)$$

Now, $1 + 3 + 5 + 7 + \cdots + (2k - 1) + \{2(k + 1) - 1\}$

$$= \{1 + 3 + 5 + 7 + \cdots + (2k - 1)\} + (2k + 1) = k^2 + (2k + 1)$$

$= (k + 1)^2$. [using (1)]

$$\therefore \quad P(k + 1): 1 + 3 + 5 + 7 + \cdots (2k - 1) + \{2(k + 1) - 1\} = (k + 1)^2.$$

This shows that $P(k + 1)$ is true, whenever $P(k)$ is true.

Thus, $P(1)$ is true and $P(k + 1)$ is true, whenever $P(k)$ is true.

Hence, by the principle of mathematical induction, we have

$$1 + 3 + 5 + 7 + \cdots + (2n - 1) = n^2 \text{ for all } n \in N.$$

Example 2: Using the principle of mathematical induction, prove that

$$1 + 4 + 7 + 10 + \cdots + (3n - 2) = \frac{1}{2}n(3n - 1) \text{ for all } n \in N.$$

Solution: Let the given statement be $P(n)$. Then,

$$P(n): 1 + 4 + 7 + 10 + \cdots + (3n - 2) = \frac{1}{2}n(3n - 1).$$

Putting $n = 1$ in the given statement, we get

$$\text{LHS} = 1 \text{ and RHS } = \frac{1}{2} \times 1 \times (3 \times 1 - 1) = 1.$$

$\therefore \quad$ LHS = RHS.
Thus, $P(1)$ is true.
Let $P(k)$ be true. Then,

$$P(k): 1 + 4 + 7 + 10 + \cdots + (3k - 2) = \frac{1}{2}k(3k - 1). \qquad (1)$$

Now, $1 + 4 + 7 + \cdots + (3k - 2) + \{3(k + 1) - 2\}$

$$= \{1 + 4 + 7 + \cdots (3k - 2)\} + (3k + 1)$$

$$= \frac{1}{2}k(3k - 1) + (3k + 1) \qquad [using\ (1)]$$

$$= \frac{1}{2}(3k^2 - k + 6k + 2) = \frac{1}{2}(3k^2 + 5k + 2)$$

$$= \frac{1}{2}(3k^2 + 3k + 2k + 2) = \frac{1}{2} \cdot \{3k(k + 1) + 2(k + 1)\}$$

$$= \frac{1}{2}(k + 1)(3k + 2) = \frac{1}{2}(k + 1)\{3(k + 1) - 1\}.$$

$$\therefore \quad P(k + 1): 1 + 4 + 7 + \cdots + \{3(k + 1) - 2\} = \frac{1}{2}(k + 1)\{3(k + 1) - 1\}.$$

This shows that $P(k + 1)$ is true, whenever $P(k)$ is true.

Thus, $P(1)$ is true and $P(k+1)$ is true, whenever $P(k)$ is true.

Hence, by the principle of mathematical induction, we have

$$1+4+7+\cdots+(3n-2)=\frac{1}{2}n(3n-1) \text{ for all } n \in N.$$

Example 3: Using the principle of mathematical induction, prove that

$$1^2+2^2+3^2+\cdots+n^2=\frac{1}{6}n(n+1)(2n+1) \text{ for all } n \in N.$$

Solution: Let the given statement be $P(n)$. Then,

$$P(n): 1^2+2^2+3^2+\cdots+n^2=\frac{1}{6}n(n+1)(2n+1).$$

Putting $n=1$ in the given statement, we get

$$\text{LHS} = 1^2 = 1 \text{ and RHS } = \frac{1}{6}\times 1\times 2\times(2\times 1+1)=1.$$

$$\therefore \quad \text{LHS} = \text{RHS}.$$

Thus, $P(1)$ is true.
Let $P(k)$ be true. Then,

$$P(k): 1^2+2^2+3^2+\cdots+k^2=\frac{1}{6}k(k+1)(2k+1).$$

Now, $1^2+2^2+3^2+\cdots+k^2+(k+1)^2$

$=\{1^2+2^2+3^2+\cdots+k^2\}+(k+1)^2=\frac{1}{6}k(k+1)(2k+1)+(k+1)^2$
[using (i)]

$$=\frac{1}{6}(k+1)\cdot\{k(2k+1)+6(k+1)\}=\frac{1}{6}(k+1)(2k^2+7k+6)$$

$$=\frac{1}{6}(k+1)\{(2k^2+4k)+(3k+6)\}$$

$$=\frac{1}{6}(k+1)(k+2)(2k+3)=\frac{1}{6}(k+1)(k+1+1)[2(k+1)+1].$$

$$\therefore \quad P(k+1): 1^2+2^2+3^2+\cdots+(k+1)^2$$

$$=\frac{1}{6}(k+1)(k+1+1)[2(k+1)+1].$$

This shows that $P(k+1)$ is true, whenever $P(k)$ is true.

Thus, $P(1)$ is true and $P(k+1)$ is true, whenever $P(k)$ is true.

Hence, by the principle of mathematical induction, we have

$$1^2+2^2+3^2+\cdots+n^2=\frac{1}{6}n(n+1)(2n+1) \text{ for all } n \in N.$$

Example 4: Using the principle of mathematical induction, prove that

$$1^3+2^3+3^3+\cdots+n^3=\left\{\frac{n(n+1)}{2}\right\}^2 \text{ for all } n \in N.$$

Solution: Let the given statement be $P(n)$. Then,

$$P(n): 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left\{\frac{n(n+1)}{2}\right\}^2.$$

Putting $n = 1$ in the given statement, we get

$$\text{LHS} = 1^3 = 1 \text{ and RHS } = \left(\frac{1 \times 2}{2}\right)^2 = 1^2 = 1.$$

$$\therefore \quad \text{LHS} = \text{RHS}.$$

Thus, $P(1)$ is true.

Let $P(k)$ be true for some $k \in N$. Then,

$$P(k): 1^3 + 2^3 + 3^3 + \cdots k^3 = \left\{\frac{k(k+1)^2}{2}\right\}^2. \quad (1)$$

Now, $1^3 + 2^3 + 3^3 + \cdots + k^3 + (k+1)^3$

$$= \{1^3 + 2^3 + 3^3 + \cdots + k^3\} + (k+1)^3$$

$$= \left\{\frac{k(k+1)}{2}\right\}^2 + (k+1)^3$$

[using (i)]

$$= (k+1)^2 \left\{\frac{k^2}{4} + (k+1)\right\} = (k+1)^2 \left\{\frac{k^2 + 4k + 4}{4}\right\}$$

$$= \frac{(k+1)^2 (k+2)^2}{4} = \left\{\frac{(k+1)\{(k+1)+1\}}{2}\right\}^2$$

$$\therefore \quad P(k+1): 1^3 + 2^3 + 3^3 + \cdots + (k+1)^3 = \left[\frac{(k+1)\{(k+1)+1\}}{2}\right]^2.$$

This shows that $P(k+1)$ is true, whenever $P(k)$ is true.

$\therefore$ $P(1)$ is true and $P(k+1)$ is true, whenever $P(k)$ is true.

Hence, by the principle of mathematical induction, we have

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left\{\frac{n(n+1)}{2}\right\}^2 \text{ for all } n \in N.$$

Example 5: Using the principle of mathematical induction, prove that

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \frac{1}{3} n(n+1)(n+2).$$

Solution: Let the given statement be $P(n)$. Then,

$$P(n): 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \frac{1}{3} n(n+1)(n+2).$$

When $n = 1$, LHS $= 1 \cdot 2 = 2$ and RHS $= \frac{1}{3} \times 1 \times 2 \times (1+2) = 2$.

$\therefore$ LHS = RHS.

Thus, the given statement is true for $n = 1$, i.e., $P(1)$ is true.

Let $P(k)$ be true. Then,

$$P(k): 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + k(k+1) = \frac{1}{3} k(k+1)(k+2). \quad (1)$$

Now, $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + k(k+1) + (k+1)(k+2)$

$$= \{1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + k(k+1) + (k+1)(k+2)$$

$$= \frac{1}{3}k(k+1)(k+2) + (k+1)(k+2) \qquad \text{[using (1)]}$$

$$= \frac{1}{3} \cdot [k(k+1)(k+2) + 3(k+1)(k+2)] = \frac{1}{3}(k+1)(k+2)(k+3).$$

$\therefore \quad P(k+1): 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + (k+1)(k+2)$

$$= \frac{1}{3}(k+1)(k+2)(k+3).$$

This shows that $P(k+1)$ is true, whenever $P(k)$ is true.

$\therefore \quad P(1)$ is true and $P(k+1)$ is true, whenever $P(k)$ is true.

Hence, by the principle of mathematical induction, we have

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \frac{1}{3}n(n+1)(n+2) \text{ for all } n \in N.$$

Example 6: Using the principle of mathematical induction, prove that

$$1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \cdots + (2n-1)(2n+1) = \frac{1}{3}n(4n^2 + 6n - 1).$$

Solution: Let the given statement be $P(n)$. Then,

$$P(n): 1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \cdots + (2n-1)(2n+1) = \frac{1}{3}n(4n^2 + 6n - 1).$$

When $n = 1$, we have

$$\text{LHS } = 1 \cdot 3 = 3 \text{ and RHS } = \frac{1}{3} \times 1 \times (4 \times 1^2 + 6 \times 1 - 1) = \frac{1}{3} \times 1 \times 9 = 3.$$

$\therefore \quad$ LHS $=$ RHS.

Thus, $P(1)$ is true.

Let $P(k)$ be true. Then,

$$P(k): 1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \cdots + (2k-1)(2k+1) = \frac{1}{3}k(4k^2 + 6k - 1). \qquad (1)$$

Now, $1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \cdots + (2k-1)(2k+1) + \{2(k+1) - 1\}\{2(k+1) + 1\}$

$$= \{1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \cdots + (2k-1)(2k+1)\} + (2k+1)(2k+3)$$

$$= \frac{1}{3}k(4k^2 + 6k - 1) + (2k+1)(2k+3)$$

[using (1)]

$$= \frac{1}{3}[(4k^3 + 6k^2 - k) + 3(4k^2 + 8k + 3)] = \frac{1}{3}(4k^3 + 18k^2 + 23k + 9)$$

$$= \frac{1}{3}(k+1)(4k^2 + 14k + 9) = \frac{1}{3}(k+1)[4(k+1)^2 + 6(k+1) - 1].$$

$\therefore \quad P(k+1): 1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \cdots + \{2(k+1) - 1\}\{2(k+1) + 1\}$

$$= \frac{1}{3}(k+1)\{4(k+1)^2 + 6(k+1) - 1\}.$$

Thus, $P(k+1)$ is true, whenever $P(k)$ is true.

$\therefore \quad P(1)$ is true and $P(k+1)$ is true, whenever $P(k)$ is true.

Hence, by the principle of mathematical induction, we have

$$1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \cdots + (2n-1)(2n+1) = \frac{1}{3}n(4n^2 + 6n - 1) \text{ for all } n \in N.$$

Example 7: Using the principle of mathematical induction, prove that

$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + n(n+1)(n+2) = \frac{1}{4}n(n+1)(n+2)(n+3)$$

for all $n \in N$.

Solution: Let the given statement be $P(n)$. Then,

$$P(n): 1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + n(n+1)(n+2) = \frac{1}{4}n(n+1)(n+2)(n+3).$$

When $n = 1$, we have

$$\text{LHS} = 1 \cdot 2 \cdot 3 = 6 \text{ and RHS } = \frac{1}{4} \times 1 \times 2 \times 3 \times 4 = 6.$$

$$\therefore \quad \text{LHS} = \text{RHS}.$$

Thus, the given statement is true for $n = 1$, i.e., $P(1)$ is true.

Let $P(k)$ be true. Then,

$$P(k): \quad 1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + k(k+1)(k+2)$$
$$= \frac{1}{4}k(k+1)(k+2)(k+3). \qquad (1)$$

Now, $1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + k(k+1)(k+2) + (k+1)(k+2)(k+3)$

$$= \{1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + k(k+1)(k+2)\} + (k+1)(k+2)(k+3)$$
$$= \frac{1}{4}k(k+1)(k+2)(k+3) + \frac{4(k+1)(k+2)(k+3)}{4} \quad \text{[using (i)]}$$
$$= \frac{1}{4}(k+1)(k+2)(k+3)(k+4).$$

[using (1)]

$$\therefore \quad P(k+1): 1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + (k+1)(k+2)(k+3)$$
$$= \frac{1}{4}(k+1)(k+2)(k+3)(k+4).$$

Thus, $P(k+1)$ is true, whenever $P(k)$ is true.

$P(1)$ is true and $P(k+1)$ is true, whenever $P(k)$ is true.

Hence, by the principle of mathematical induction, we have

$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + n(n+1)(n+2) = \frac{1}{4}n(n+1)(n+2)(n+3)$$

for all values of $n \in N$.

Example 8: Using the principle of mathematical induction, prove that

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{(n+1)}.$$

Solution: Let the given statement be $P(n)$. Then,

$$P(n): \frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{n(n+1)}=\frac{n}{(n+1)}.$$

Putting $n=1$ in the given statement, we get

$$\text{LHS} = \frac{1}{1\cdot 2}=\frac{1}{2} \text{ and RHS } =\frac{1}{(1+1)}=\frac{1}{2}.$$

$$\therefore \quad \text{LHS} = \text{RHS}.$$

Thus, $P(1)$ is true.

Let $P(k)$ be true. Then,

$$P(k): \frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{k(k+1)}=\frac{k}{(k+1)}. \tag{1}$$

Now, $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)}$

$$=\left\{\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{k(k+1)}\right\}+\frac{1}{(k+1)(k+2)}$$

$$=\frac{k}{(k+1)}+\frac{1}{(k+1)(k+2)}$$

[using (1)]

$$=\frac{k(k+2)+1}{(k+1)(k+2)}=\frac{(k+1)^2}{(k+1)(k+2)}=\frac{(k+1)}{(k+2)}.$$

$\therefore \quad P(k+1): \frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}=\frac{1}{(k+1)(k+2)}=\frac{(k+1)}{(k+2)}.$

Thus, $P(k+1)$ is true, whenever $P(k)$ is true.

$\therefore \quad P(1)$ is true and $P(k+1)$ is true, whenever $P(k)$ is true.

Hence, by the principle of mathematical induction, we have

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{n(n+1)}=\frac{n}{n+1} \text{ for all } n\in N.$$

Example 9: Using the principle of mathematical induction, prove that

$$\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\frac{1}{7\cdot 9}+\cdots+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}.$$

Solution: Let the given statement be $P(n)$. Then,

$$P(n): \frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\frac{1}{7\cdot 9}+\cdots+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}.$$

Putting $n=1$ in the given statement, we get

$$\text{LHS} = \frac{1}{3\cdot 5}=\frac{1}{15} \text{ and RHS } =\frac{1}{3(2\times 1+3)}=\frac{1}{15}.$$

$$\therefore \quad \text{LHS} = \text{RHS}.$$

Thus, $P(1)$ is true.

Let $P(k)$ be true. Then,

$$P(k): \frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\frac{1}{7\cdot 9}+\cdots+\frac{1}{(2k+1)(2k+3)}=\frac{k}{3(2k+3)}. \quad (1)$$

Now, $\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots+\frac{1}{(2k+1)(2k+3)}+\frac{1}{\{2(k+1)+1\}\{2(k+1)+3\}}$

$$=\left\{\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots+\frac{1}{(2k+1)(2k+3)}\right\}+\frac{1}{(2k+3)(2k+5)}$$

$$=\frac{k}{3(2k+3)}+\frac{1}{(2k+3)(2k+5)} \quad \text{[using (1)]}$$

$$=\frac{k(2k+5)+3}{3(2k+3)(2k+5)}=\frac{(2k^2+5k+3)}{3(2k+3)(2k+5)}=\frac{(k+1)(2k+3)}{3(2k+3)(2k+5)}$$

$$=\frac{(k+1)}{3(2k+5)}=\frac{(k+1)}{3\{2(k+1)+3\}}.$$

$$\therefore \quad P(k+1): \frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\frac{1}{7\cdot 9}+\cdots+\frac{1}{\{2(k+1)+1\}\{2(k+1)+3\}}$$
$$=\frac{(k+1)}{3\{2(k+1)+3\}}.$$

This shows that $P(k+1)$ is true, whenever $P(k)$ is true. $\therefore$ $P(1)$ is true and $P(k+1)$ is true, whenever $P(k)$ is true. Hence, by the principle of mathematical induction, we have

$$\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\frac{1}{7\cdot 9}+\cdots+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}$$

for all values of $n \in N$.

Example 10: Using the principle of mathematical induction, prove that

$$\frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\cdots+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)} \text{ for all } n \in N.$$

Solution: Let the given statement be $P(n)$. Then,

$$P(n): \frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\cdots+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}.$$

Putting $n=1$ in the given statement, we get

$$\text{LHS } =\frac{1}{1\cdot 2\cdot 3}=\frac{1}{6} \text{ and RHS } =\frac{1\times(1+3)}{4\times(1+1)(1+2)}=\frac{1\times 4}{4\times 2\times 3}=\frac{1}{6}.$$

$$\therefore \quad \text{LHS } = \text{ RHS}.$$

Thus, the given statement is true for $n=1$, i.e., $P(1)$ is true.

Let $P(k)$ be true. Then,

$$P(k): \frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\cdots+\frac{1}{k(k+1)(k+2)}=\frac{k(k+3)}{4(k+1)(k+2)}. \quad (1)$$

Now, $\frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\cdots+\frac{1}{k(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)}$

$$= \left\{\frac{1}{1\cdot 2\cdot 3} + \frac{1}{2\cdot 3\cdot 4} + \cdots + \frac{1}{k(k+1)(k+2)}\right\} + \frac{1}{(k+1)(k+2)(k+3)}$$

$$= \left\{\frac{k(k+3)}{4(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)}\right\} \quad \text{[using (1)]}$$

$$= \frac{k(k+3)^2 + 4}{4(k+1)(k+2)(k+3)} = \frac{(k^3 + 6k^2 + 9k + 4)}{4(k+1)(k+2)(k+3)}$$

$$= \frac{(k+1)(k+1)(k+4)}{4(k+1)(k+2)(k+3)} = \frac{(k+1)(k+4)}{4(k+2)(k+3)}.$$

$$P(k+1): \frac{1}{1\cdot 2\cdot 3} + \frac{1}{2\cdot 3\cdot 4} + \cdots + \frac{1}{(k+1)(k+2)(k+3)}$$

$$\therefore \quad = \frac{(k+1)(k+4)}{4(k+2)(k+3)}.$$

This shows that $P(k+1)$ is true, whenever $P(k)$ is true. $\therefore$ $P(1)$ is true and $P(k+1)$ is true, whenever $P(k)$ is true.

Hence, by the principle of mathematical induction, we have

$$\frac{1}{1\cdot 2\cdot 3} + \frac{1}{2\cdot 3\cdot 4} + \frac{1}{3\cdot 4\cdot 5} + \cdots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

for all values of $n \in N$.

2.4 LET US SUM UP

So, in this unit we have learned about the Principle of Mathematical Induction. We also discussed various examples to explore the induction.

2.5 UNIT END EXERCISE

By using principle of mathematical induction show that.

2.5.1 $1^4 + 2^4 + 3^4 + \cdots + n^4 = n(n+1)(2n+1)\frac{(3n^2+3n-1)}{30}$.

2.5.2 $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$.

2.5.3 $1 + r + r^2 + \cdots + r^n = \frac{1-r^{n+1}}{1-r}, \forall\, n \in \mathbb{N}, (r \neq 1)$.

2.5.4 $7^n - 4^n$ is a multiple of 3 for all $n \in \mathbb{N}$.

2.5.5 $5^{2n} - 1$ is a multiple of 8.

2.5.6 $9^n - 4^n$ is a multiple of 5.

2.6 REFERENCES AND SUGGESTED READINGS

I. David Burton, M. (2007). ELEMENTARY NUMBER THEORY Seventh Edition.

II. Rosen, K. H. (2011). *Elementary number theory*. London: Pearson Education.

III. Hari Kishan. (2014). Number Theory. Krishna Prakashan, Meerut.

IV. Hari Kishan. Shiv Raj Pundir. Discrete Mathematics. Pragati Prakashan, Meerut.

***