BACHELOR OF COMPUTER APPLICATIONS (BCA)

CSB-1113 (Mathematics - I)

Block 2: Introduction to Matrix Theory

Unit 8: System of Linear Equations

Structure

8.1 INTRODUCTION

Welcome to the unit on Systems of Linear Equations! In this module, we will embark on a journey to explore the fundamental concepts and techniques essential for understanding and solving systems of linear equations, which are prevalent in various fields ranging from mathematics and physics to engineering and economics.

Understanding the Significance:

A system of linear equations represents a collection of equations involving multiple variables, where each equation is linear. These systems serve as powerful tools for modeling real-world scenarios and finding solutions to complex problems. Whether it's determining the intersection of lines in geometry, analyzing electrical circuits, or optimizing production processes, systems of linear equations offer invaluable insights.

8.2OBJECTIVES

After completing this unit, students will be able to

- Write system of linear equations in matrix form
- Differentiate homogeneous and non-homogeneous system
- Decide whether a non-homogeneous system has a solution or not
- Find the solution of system

8.3SYSTEM OF LINEAR EQUATIONS

An equation of first degree in n unknowns $x_1, x_2, x_3, \ldots.., x_n$ is called a linear equation.
Thus,

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots. + a_{1n}x_n = b_1$$

is a linear equation in n unknowns $x_1, x_2, x_3, \ldots.., x_n$ with coefficients $a_{11}, a_{12}, a_{13}, \ldots.., a_{1n}$ and b_1 as constants.

If $b_1 = 0$, then Eq. (6.1) takes the form

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots.. + a_{1n}x_n = 0$$

and is called a homogeneous linear equation in n unknowns $x_1, x_2, x_3, \ldots \ldots, x_n$.

Thus,

$2x + 3y + 4z = 20$ is an example of a non-homogeneous linear equation in 3 unknowns x, y and z

whereas $x - y + z = 0$ is an example of a homogeneous linear equation in 3 unknowns x, y and z.

System of Linear Equations

Consider a system of m linear equations in n unknowns ($> n, m = n$ or $m < n$) given below:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots. + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots.. + a_{2n}x_n &= b_2 \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots. + a_{mn}x_n &= b_m \end{aligned}$$

In matrix notation, these equations can be put in the form

$$AX = B$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \ldots.. & a_{1n} \\ a_{21} & a_{22} & a_{23} & \ldots. & a_{2n} \\ \ldots.. & \ldots.. & \ldots.. & \ldots. & \ldots.. \\ \ldots.. & \ldots.. & \ldots.. & \ldots. & \ldots\ldots \\ a_{m1} & a_{m2} & a_{m3} & \ldots. & a_{mn} \end{bmatrix}_{m\times n}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{m\times 1} \quad \text{and } B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ \vdots \\ b_m \end{bmatrix}_{m\times 1}$$

The matrix A is called the coefficient matrix. The matrix X is called the column matrix of n unknowns $x_1, x_2, x_3, \ldots, x_n$. The matrix B is called the column matrix of m constants $b_1, b_2, \ldots.., b_m$.

The matrix $C = [A:B]$ obtained by placing the constant column matrix B to the right of the matrix A is called augmented matrix. Thus, the matrix

$$C = [A:B] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \ldots.. & a_{1n} & \vdots & b_1 \\ a_{21} & a_{22} & a_{23} & \ldots.. & a_{2n} & \vdots & b_2 \\ \ldots.. & \ldots.. & \ldots. & \ldots.. & \ldots.. & \vdots & \ldots \\ \ldots.. & \ldots.. & \ldots.. & \ldots.. & \ldots.. & \vdots & \ldots \\ a_{m1} & a_{m2} & a_{m3} & \ldots.. & a_{mn} & \vdots & b_m \end{bmatrix}$$

is called the augmented matrix.

Any set of values of x_1, x_2, x_3, x_n which simultaneously satisfy the system of equation (A) is called the solution of the system (A). If the system has one or more solutions, it is called consistent. If it has no solution, it is called inconsistent. A consistent system has either one solution or infinitely many solutions.

Example:

(i) The system of equations

$$\begin{aligned} x_1 + x_2 &= 3 \\ 2x_1 + 3x_2 &= 8 \end{aligned}$$

is consistent because it has a unique solution $x_1 = 1$,

$$x_2 = 2.$$

(ii) The system of equations

$$\begin{aligned} 2x_1 + 3x_2 &= 5 \\ 6x_1 + 9x_2 &= 15 \end{aligned}$$

is consistent because it has infinitely many solutions. Note that here the two equations are one and the same.

(iii) The system of equations

$$\begin{aligned} x_1 - 4x_2 &= 2 \\ 2x_1 - 8x_2 &= 5 \end{aligned}$$

is inconsistent because it has no solution.

Non-singular or Regular System of Linear Equations

If we take $m = n$ in the system of equations (A), then we have

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \ldots.. & a_{1n} \\ a_{21} & a_{22} & a_{23} & \ldots\ldots & a_{2n} \\ \ldots\ldots & \ldots\ldots & \ldots.. & \ldots\ldots & \ldots\ldots \\ \ldots\ldots & \ldots\ldots & \ldots.. & \ldots.. & \ldots\ldots \\ a_{n1} & a_{n2} & a_{n3} & \ldots.. & a_{nn} \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} & \ldots.. & a_{1n} \\ a_{21} & a_{22} & a_{23} & \ldots.. & a_{2n} \\ \ldots.. & \ldots.. & \ldots.. & \ldots.. & \ldots.. \\ \ldots.. & \ldots\ldots & \ldots\ldots\ldots. & & \ldots\ldots \\ a_{n1} & a_{n2} & a_{n3} & \ldots.. & a_{nn} \end{vmatrix}$$

$|A|$ is called the determinant of coefficients.

If $|A| \neq 0$, then the system of equations (when $m = n$) is called regular. This system has a unique solution given by

$\frac{x_1}{|A_1|} = \frac{x_2}{|A_2|} = \frac{x_3}{|A_3|} = \cdots.. = \frac{x_n}{|A_n|} = \frac{1}{|A|}$ where $|A_1|$ represents the determinant obtained by replacing the r^{th} column of $|A|$ by the column of b 's.

This is known as Cramer's Rule for solving a system of n linear equation in n unknowns.

Note. If $|A| = 0$, then Cramer's Rule fails.

When $|A| \neq 0$, then A^{-1} exists. Hence, pre-multiplying the matrix equation

$$AX = B$$

by A^{-1}, we obtain,

$$\begin{aligned} & A^{-1}(AX) = A^{-1}B \\ \Rightarrow \quad & (A^{-1}A)X = A^{-1}B \\ \Rightarrow \quad & IX = A^{-1}B \\ \Rightarrow \quad & X = A^{-1}B \end{aligned}$$

This gives the solution of the system of n equations in n unknowns when the system is non-singular (or regular).

Singular System of Linear Equations

If $|A| = 0$, the system of n equations in n unknowns is called singular. This case will be dealt with later on.

System of Linear Equations, in General

Consider the system of linear equations (A)

It is simple to see that the rank of the augmented matrix C cannot be less than the rank of the coefficient matrix A because every sub-matrix of A is also a sub-matrix of C.

Let $\rho(A) = r$. Then, by a suitable sequence of elementary row operations, the matrix A can be reduced to an equivalent matrix in which each of the first r elements of the leading diagonal is 1 and every element below this diagonal and/or below the r^{th} row is zero. The matrix, so reduced, is said to be in Echelon Form.

If the same sequence of elementary operations is performed on the system of equations (A), then these will be transformed in the following form:

$$\begin{aligned}
&y_1 + \alpha_{12}y_2 + \alpha_{13}y_3 + \cdots. + \alpha_{1n}y_n = \beta_1 \\
&y_2 + \alpha_{23}y_3 + \cdots. + \alpha_{2n}y_n = \beta_2 \\
&y_r + \cdots.. + \alpha_{m}y_n = \beta \\
&0 = \beta_{r+1} \\
&\dots\dots = \cdots.. \\
&\dots\dots = \cdots. \\
&0 = \beta_m
\end{aligned}$$

in which $y_1, y_2, \dots\dots, y_n$ is some permutation of $x_1, x_2, \dots\dots, x_n$.

Also, the coefficient matrix and the augmented matrix of the system of equations (B) will be equivalent to A and C respectively.

Now the following cases arise:

Case I: When $r = 1$, then the system of equations (B) becomes,

$$\left.\begin{aligned}
y_1 + \alpha_{12}y_2 + \alpha_{13}y_3 + \cdots.. + \alpha_{1n}y_n &= \beta_1 \\
0 &= \beta_1 \\
0 &= \beta_3 \\
&\vdots \\
&\vdots \\
0 &= \beta_m
\end{aligned}\right\} \dots \text{(C)}$$

In this case, $\rho(C) = 2$ or 1 , since C has one column more than A.

When $\rho(C) = 2, \beta_2, \beta_3, \dots., \beta_m$ cannot all be zero. Hence, the equations are inconsistent and there will be no solution.

When $\rho(C) = 2, \beta_2, \beta_3, \dots.., \beta_m$ will be all zero and the system of equations (B) will be equivalent to a single equation from which y_1 will be expressible in terms of $y_2, y_3, \dots.., y_n$ which can have arbitrary values.

Case D. When $r = 2$, then the system of equations (B) becomes

$$\left.\begin{array}{rl} y_1 + \alpha_{12}y_2 + \alpha_{13}y_3 + \cdots. + \alpha_{1n}y_n & = \beta_1 \\ y_2 + \alpha_{23}y_3 + \cdots. + \alpha_{2n}y_n & = \beta_2 \\ 0 & = \beta_3 \\ 0 & = \beta_4 \\ & \vdots \\ & \vdots \\ 0 & = \beta_m \end{array}\right\} \ldots$$

In this case, $\rho(C) = 3$ or 2

When $\rho(C) = 3, \beta_3, \beta_4, \ldots\ldots, \beta_m$ cannot all be zero. Hence, the equations are inconsistent and there will be no solution.

But when $\rho(C) = 2, \beta_3, \beta_4, \ldots\ldots, \beta_m$ will be all zero and the equations will be equivalent to two independent equations from which y_1 and y_2 will be expressible in terms of y_3, y_4, ..., y_n which can have arbitrary values.

Similarly, when $r = 3$, then $\rho(C)$ must also be 3 in order that the equations may be consistent and in that case y_1, y_2, y_3 will be expressible in terms of $y_4, y_5, \ldots, y_n$ which are arbitrary.

In general, the necessary and sufficient conditions that the equations (B) may be consistent is that $\rho(C) = \rho(A)$, i.e. if the coefficient matrix A and the augmented matrix C have the same rank and if each rank $= r$, the equations will be equivalent to r equations from which r unknowns can be

Example 1: Solve, with the help of matrices, the simultaneous equations:

$$x + y + z = 3$$
$$x + 2y + 3z = 4$$
$$x + 4y + 9z = 6$$

Solution:

Coefficient Matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$

$$\begin{aligned} \therefore |A| &= \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{vmatrix} \\ &= 1(18 - 12) + 1(3 - 9) + 1(4 - 2) \\ &= 6 - 6 + 2 \\ &= 2 \neq 0 \\ \therefore \rho(A) &= 3 \end{aligned}$$

Augmented matrix $[A{:}B] = \begin{bmatrix} 1 & 1 & 1 & \vdots & 3 \\ 1 & 2 & 3 & \vdots & 4 \\ 1 & 4 & 9 & \vdots & 6 \end{bmatrix}$

Operating $\mathrm{R}_2 - R_1, \mathrm{R}_3 - \mathrm{R}_1$

$$- \begin{bmatrix} 1 & 1 & 1 & \vdots & 3 \\ 0 & 1 & 2 & \vdots & 1 \\ 0 & 3 & 8 & \vdots & 3 \end{bmatrix}$$

Operating $R_3 - 3R_1$

$$- \begin{bmatrix} 1 & 1 & 1 & \vdots & 3 \\ 0 & 1 & 2 & \vdots & 1 \\ 0 & 0 & 2 & \vdots & 0 \end{bmatrix}$$

$$\therefore \rho[A{:}B] = 3$$
$$\because \rho \mid A{:}B] = \rho[A] = 3 \text{ (number of unknowns)}$$

∴ The given system of equations is consistent and has a unique solution.

Equivalent system of equations is

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad x + y + z = 3$$
$$y + 2z = 1$$
$$2z = 0$$
$$\Rightarrow \quad x = 2, y = 1, z = 0$$

Example 2: Solve the system of equations by using matrix method:

$$2x_1 + x_2 + 2x_3 + x_4 = 6$$
$$6x_1 - 6x_2 + 6x_3 + 12x_4 = 36$$
$$4x_1 + 3x_2 + 3x_3 - 3x_4 = -1$$
$$2x_1 + 2x_2 - x_3 + x_4 = 10$$

Solution:

$$\text{Coefficient matrix } A = \begin{bmatrix} 2 & 1 & 2 & 1 \\ 6 & -6 & 6 & 12 \\ 4 & 3 & 3 & -3 \\ 2 & 2 & -1 & 1 \end{bmatrix}$$

$$\because |A| \neq 0$$
$$\therefore \rho(A) = 4$$

$$\text{Augmented matrix } [A:B] = \begin{bmatrix} 2 & 1 & 2 & 1 & \vdots & 6 \\ 6 & -6 & 6 & 12 & \vdots & 36 \\ 4 & 3 & 3 & -3 & \vdots & -1 \\ 2 & 2 & -1 & 1 & \vdots & 10 \end{bmatrix}$$

Equivalent system of equations is

$$\begin{bmatrix} 2 & 1 & 2 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & -4 \\ 0 & 0 & 0 & 13 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \\ -11 \\ 39 \end{bmatrix}$$

$$\Rightarrow 2x_1 + x_2 + 2x_3 + x_4 = 6$$
$$x_2 - x_4 = -2$$
$$-x_3 - 4x_4 = -11$$
$$13x_4 = 39$$

On solving, we get

$$x_1 = 2, x_2 = 1, x_3 = -1, x_4 = 3$$

Example 3: Using matrix method, show that the equations

$$3x + 3y + 2z = 1$$
$$x + 2y = 4$$
$$10y + 3z = -2$$
$$2x - 3y - z = 5$$

are consistent and hence obtain the solutions for x, y and z.

Solution:

$$\text{Coefficient matrix } A = \begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 0 \\ 0 & 10 & 3 \\ 2 & -3 & -1 \end{bmatrix}$$

$$\because \text{ Minor } \begin{vmatrix} 1 & 2 & 0 \\ 0 & 10 & 3 \\ 2 & -3 & -1 \end{vmatrix} \neq 0$$

$$\therefore \rho(A) = 3$$

$$\therefore \rho[A:B] = 3$$

$$\because \rho[A:B] = \rho[A] = 3 \text{ (no. of variables)}$$

$\therefore$ The given system of equations is consistent and has a unique solution.

Equivalent system of equations is

$$\begin{bmatrix} 1 & 0 & -18 \\ 0 & 1 & 9 \\ 0 & 0 & 29 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 74 \\ -35 \\ -116 \\ 0 \end{bmatrix}$$

$$\Rightarrow x - 18z = 74$$

$$y + 9z = -35$$

$$29z = -116$$

$$\Rightarrow x = 2, y = 1, z = -4$$

Example 4: Show that the following system of equations is inconsistent:

$$x_1 - 2x_2 + x_3 - x_4 + 1 = 0$$

$$3x_1 - 2x_3 + 3x_4 + 4 = 0$$

$$5x_1 - 4x_2 + x_4 + 3 = 0$$

Solution:

$$\text{Coefficient matrix } A = \begin{bmatrix} 1 & -2 & 1 & -1 \\ 3 & 0 & -2 & 3 \\ 5 & -4 & 0 & 1 \end{bmatrix}$$

$$\text{Augmented matrix } [A:B] = \begin{bmatrix} 1 & -2 & 1 & -1 & \vdots & -1 \\ 3 & 0 & -2 & 3 & \vdots & -4 \\ 5 & -4 & 0 & 1 & \vdots & -3 \end{bmatrix}$$

Operating $R_2 - 3R_1, R_3 - 5R_1$

$$- \begin{bmatrix} 1 & -2 & 1 & -1 & \vdots & -1 \\ 0 & 6 & -5 & 6 & \vdots & -1 \\ 0 & 6 & -5 & 6 & \vdots & 2 \end{bmatrix}$$

Operating $R_3 - R_1$

$$- \begin{bmatrix} 1 & -2 & 1 & -1 & \vdots & -1 \\ 0 & 6 & -5 & 6 & \vdots & -1 \\ 0 & 0 & 0 & 0 & \vdots & 3 \end{bmatrix}$$

which is Echelon Form.

$$\text{Clearly, } \rho(A) = 2$$

$$\rho(\mathrm{A}:B) = 3$$

$$\therefore \rho(A) \neq \rho(\mathrm{A}:B)$$

Hence, the given system of equations is inconsistent.

Example 5: Investigate for consistency of the following equations and if possible, find the solutions.

$$4x - 2y + 6z = 8$$
$$x + y - 3z = -1$$
$$15x - 3y + 9z = 21$$

Solution:

Coefficient matrix $A = \begin{bmatrix} 4 & -2 & 6 \\ 1 & 1 & -3 \\ 15 & -3 & 9 \end{bmatrix}$

Operating R_{12}

$$- \begin{bmatrix} 1 & 1 & -3 & \vdots & -1 \\ 4 & -2 & 6 & \vdots & 8 \\ 15 & -3 & 9 & \vdots & 21 \end{bmatrix}$$

Operating $R_2 - 4\mathrm{R}_1, R_3 - 15\mathrm{R}_1)$

$$- \begin{bmatrix} 1 & 1 & -3 & \vdots & -1 \\ 0 & -6 & 18 & \vdots & 12 \\ 0 & -18 & 54 & \vdots & 36 \end{bmatrix}$$

Operating $R_3 - 3\mathrm{R}_2$

$$- \begin{bmatrix} 1 & 1 & -3 & \vdots & -1 \\ 0 & -6 & 18 & \vdots & 12 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$$

which is Echelon Form.
Clearly, $\rho(A) = 2 = \rho[A: B] <$ (no. of variables)

Hence, the given system of equations is consistent and has an infinite number of solutions.

Example 6: For what values of k, the equations $x + y + z = 1, 2x + y + 4z = k$, $4x + y + 10z = k^2$ has a solution?

Solution: Here, we have

$$\begin{aligned} x + y + z &= 1 \\ 2x + y + 4z &= k \\ 4x + y + 10z &= k^2 \end{aligned}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 4 \\ 4 & 1 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ k \\ k^2 \end{bmatrix}$$

$AX = B$

$$C \quad = [A: B] = \begin{bmatrix} 1 & 1 & 1 & \vdots & 1 \\ 2 & 1 & 4 & \vdots & k \\ 4 & 1 & 10 & \vdots & k^2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & \vdots & 1 \\ 0 & -1 & 2 & \vdots & k-2 \\ 0 & -3 & 6 & \vdots & k^2 - 4 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 4R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & \vdots & 1 \\ 0 & -1 & 2 & \vdots & k-2 \\ 0 & 0 & 0 & \vdots & k^2 - 3k + 2 \end{bmatrix} \underset{R_3 \to R_3 - 3R_2}{\Rightarrow} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ k-2 \\ k^2 - 3k + 2 \end{bmatrix}$$

If the given system has solutions, then $R(A) = R(C)$ But $R(A) = 2$

$$R(C) = 2 \text{ if } k^2 - 3k + 2 = 0 \Rightarrow (k-1)(k-2) = 0 \Rightarrow k = 1, k = 2$$

Case I: When $k = 1$, we have

$$x + y + z = 1 \tag{1}$$

and

$$-y + 2z = -1 \tag{2}$$

Let$z = \lambda$. Putting the value of $z = \lambda$ in (2), we have $y + 2\lambda = -1 \Rightarrow y = 2\lambda + 1$
Putting the values of y and z in (1), we have

$$x + (2\lambda + 1) + \lambda = 1 \Rightarrow x = -3$$

Hence solution is

$$x = -3\lambda$$
$$y = 2\lambda + 1$$
$$z = \lambda$$

(is an arbitray constant)
Case II. When $k = 2$, we have

$$x + y + z = 1 \quad (3)$$

and

$$-y + 2z = 0 \quad (4)$$

Putting the value of $z = c$ in (4), we have

$$-y + 2c = 0 \Rightarrow y = 2c$$

Putting the values of y and z in (1), we have

$$x + 2c + c = 1 \Rightarrow x = -3c + 1$$

Hence the solution is

$x = 1 - 3c, y = 2c, z = c$, where c is an arbitrary constant.

Example 7: Investigate the values of λ and μ so that the equations:

$$\begin{aligned} 2x + 3y + 5z &= 9 \\ 7x + 3y - 2z &= 8 \\ 2x + 3y + \lambda z &= \mu \end{aligned}$$

have (i) no solution
(ii) a unique solution
(iii) an infinite number of solutions.

Solution: Here, we have,

$$2x + 3y + 5z = 9$$
$$7x + 3y - 2z = 8$$
$$2x + 3y + \lambda z = \mu$$

The above equations are written in the matrix form

$$\begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ \mu \end{bmatrix}$$

$$C = [A:B] = \begin{bmatrix} 2 & 3 & 5 & \vdots & 9 \\ 7 & 3 & -2 & \vdots & 8 \\ 2 & 3 & \lambda & \vdots & \mu \end{bmatrix} = \begin{bmatrix} 2 & 3 & 5 & \vdots & 9 \\ 0 & -\frac{15}{2} & -\frac{39}{2} & \vdots & -\frac{47}{2} \\ 0 & 0 & \lambda - 5 & \vdots & \mu - 9 \end{bmatrix} \begin{matrix} R_2 \to R_2 - \frac{7}{2}R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

(i) No solution. Rank $(A) \neq$ Rank (C)

$$\lambda - 5 = 0 \text{ or } \lambda = 5 \text{ and } \mu - 9 \neq 0$$

$$\Rightarrow \mu \neq 9$$

(ii) A unique solution. Rank $(A) = R(C) =$ Number of unknowns

$$\lambda - 5 \neq 0 \quad \Rightarrow \quad \lambda \neq 5 \text{ and } \mu \neq 9$$

(iii) An infinite number of solutions. Rank $(A) =$ Rank $(C) = 2$

$$\lambda - 5 = 0 \text{ and } \mu - 9 = 0$$

$$\lambda = 5 \text{ and } \mu = 9$$

8.4 LET US SUM UP

So, in this first unit we have learned about

- system of linear equations in matrix form
- Differentiate between homogeneous and non-homogeneous system
- whether a non-homogeneous system has a solution or not
- the calculation of the solution of system

8.5UNIT END EXERCISE

Solve by matrix method:

(a) $x_1 - x_2 + x_3 = 2$

$$3x_1 - x_2 + 2x_3 = -6$$
$$3x_1 + x_2 + x_3 = -18$$

(b)

$$x + 2y + 3z = 14$$
$$3x + y + 2z = 11$$
$$2x + 3y + z = 11$$

(c)

$$2x - y + 3z = 9$$
$$x + y + z = 6$$
$$x - y + z = 2$$

(d)

$$x + y + z = 6$$
$$x + 2y + 3z = 14$$
$$x + 4y + 9z = 36$$

(c)

$$x_1 + x_2 + x_3 = 0$$
$$2x_1 + 5x_2 + 6x_3 = 0$$

(f)

$$x + y + z = 6$$
$$x - y + z = 2$$
$$2x + y - z = 1$$
$$x + y + z = 3$$
$$x + 2y + 3z = 4$$
$$x + 4y + 9z = 6$$

(g)

$$\begin{aligned}x+y+z&=3\\x+2y+3z&=4\\x+4y+9z&=6\end{aligned}$$

(h)

$$\begin{aligned}x+2y+3z&=4\\2x+3y+8z&=7\\x-y+9z&=1\end{aligned}$$

(i)

$$\begin{aligned}4x+2y+z+34&=0\\6x+3y+4z+74&=0\\2x+y+4&=0\end{aligned}$$

(j)

$$\begin{aligned}x_1+x_2+x_3+x_4&=1\\2x_1-x_2+x_3-2x_4&=2\\3x_1+2x_2-x_3-x_4&=3\end{aligned}$$

(k)

$$\begin{aligned}7x-4y&=12\\-4x+12y-6z&=0\\-6y+14z&=0\end{aligned}$$

8.6 REFERENCES AND SUGGESTED READINGS

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