MANGALAYATAN UNIVERSITY
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NAAC GRADE A+ Accredited University

BACHELOR OF COMPUTER APPLICATIONS (BCA)

CSB-1113 (Mathematics - I)

Block 4: Integration

Unit 15: Method of Integration by Parts

Structure

15.1 INTRODUCTION

In the vast landscape of calculus, integration by parts stands as a crucial technique for solving integrals that do not readily yield to other methods. This unit serves as an introductory exploration into the method of integration by parts, offering learners a comprehensive understanding of its principles, applications, and limitations.

Integration by parts offers a powerful tool for dissecting complex integrals, enabling mathematicians, scientists, and engineers to solve a myriad of problems across different disciplines. Through this unit, learners will acquire a solid grasp of integration by parts, equipping them with the skills to tackle challenging integrals with confidence and proficiency.

15.2 OBJECTIVES

After completing this unit, students will be able to

1. Understand the concept of integration by parts and its significance in calculus.
2. Learn the derivation of the integration by parts formula and its intuitive interpretation.
3. Master the technique of integration by parts through guided examples and practice problems.
4. Explore strategies for choosing appropriate functions to apply integration by parts effectively.
5. Apply integration by parts to solve a variety of integrals, including those involving logarithmic, exponential, algebraic, and trigonometric functions.

15.3 METHOD OF INTEGRATION BY PARTS

Let $u = f(x)$ and $w = g(x)$ be tivo differentiable functions of x.

$\therefore \quad \frac{d}{dx}(u\omega) = u\frac{d\omega}{dx} + u\frac{du}{dx}$ or $u\frac{dw}{dx} = \frac{d}{dx}(u\omega) - \omega\frac{du}{dx}$

$$\therefore \quad \int \left(u\frac{dw}{dx}\right)dx = \int \left(\frac{d}{dx}(ute)\right)dx - \int \left(u\frac{du}{dx}\right)dx \tag{1}$$

$$\frac{dw}{dx} = v \Rightarrow w = \int \quad vdx$$

$$\therefore (1) \Rightarrow \int uvdx = \int \left(\frac{d}{dx}\left(u\int vdx\right)\right)dx - \int \left(\frac{du}{dx}\int vdx\right)dx$$

$$\therefore \quad \int uvdx = u\int vdx - \int \left(\frac{du}{dx}\int vdx\right)dx.$$

In words: Integral of the product of two functions
= 1 st function × Integral of 2 nd-Integral of [Diff, coeff. of 1st × Integral of 2nd].

From the above formula, it is clear that the success of the method of integration by parts depends on the choice of first function. For the choice of first function, the following rules are used:

Rule (i) If the integrand contains both functions integrable, take that function as the first which can be finished by repeated differentiation and the other as the second.

For example, in $\int x^2 \sin\ xdx$ or in $\int x^2 e^x dx$, take x^2 as the first function and the other as the second.

Rule (ii) If the integrand contains one unintegrable, eg., inverse circular function or a logarithmic function, take unintegrable as the first function and other as socond. For example, in $\int x^2 \log\ xdx, \int x\sin^{-1}\ xdx$, tako $\log\ x, \sin^{-1}\ x$an the first function and other as the second respectively.

Rule (iii) If the integrand contains only ons unintegrable function, then take the given function as the first function and unity as the second. For example, in $\int \tan^{-1}\ xdx$

C. $\log\left(x^2+a^2\right)dx$, take $\tan^{-1}\ x, \log\left(x^2+a^2\right)$ as the first function and unity as the second function respectively.

Rule (iv) If the integrand contains both the funetions integrable and none can be finished by repeated differentiation, then take any one as the first function and other as the second one. Repeat the rule of integration by parts.

Remark 1: If the integrand is the product of two functions of different typee then their order is determined by the word I.LATE where the letters I, **L**, **A**, **T** and **E** stands respectively for inverse circular function, logarithmic function, algobraic function, trigonometric function and exponeatial function. In the integrand, that function is taken as the first function which comes firat in the word IL.ATE.

Remark 2: Sometimes, the formula of integration by parts is required to be applied more than once in evaluating the same integral. For example, in evaluating $\int x^2 e^x dx$, the formula for integration by parts would bo used twice.

Integration by Parts
Theorem: If u and v are two functions of x then

$$\int \quad (uv)dx = \left[u \cdot \int \quad vdx\right] - \int \quad \left\{\frac{du}{dx} \cdot \int \quad vdx\right\}dx.$$

Proof: For any two functions $f_1(x)$ and $f_2(x)$, we have

$$\frac{d}{dx}[f_1(x)\cdot f_2(x)] = f_1(x)\cdot f_2{}'(x) + f_2(x)\cdot f_1{}'(x).$$

$$\therefore \quad \int \{f_1(x)\cdot f_2{}'(x) + f_2(x)\cdot f_1{}'(x)\}dx = f_1(x)\cdot f_2(x)$$

$$\text{or} \quad \int f_1(x)\cdot f_2{}'(x)dx + \int f_2(x)\cdot f_1{}'(x)dx = f_1(x)\cdot f_2(x)$$

$$\text{or} \quad \int f_1(x)\cdot f_2{}'(x)dx = f_1(x)\cdot f_2(x) - \int f_2(x)\cdot f_1{}'(x)dx.$$

Let $f_1(x) = u$ and $f_2{}'(x) = v$ so that $f_2(x) = \int v dx$.

$$\therefore \quad \int (uv)dx = u\cdot\int v dx - \int \left\{\frac{du}{dx}\cdot\int v dx\right\}dx.$$

We can express this result as given below:
Integral of product of two functions

= (1 st function) × (integral of 2 nd)

$-\int$ {(derivative of 1 st) × (integral of 2 nd)}dx.

Remarks:

(i) If the integrand is of the form $f(x)x^n$, we consider x^n as the first function and $f(x)$ as the second function.

(ii) If the integrand contains a logarithmic or an inverse trigonometric function, we take it as the first function. In all such cases, if the second function is not given, we take it as 1.

Example 1: Evaluate $\int x\sec^2 x dx$

Solution: Integrating by parts, taking x as the first function, we get

$$\begin{aligned}\int x\sec^2 x dx &= x\cdot\int \sec^2 x dx - \int\left\{\frac{d}{dx}(x)\cdot\int \sec^2 x dx\right\}dx \\ &= x\tan x - \int 1\cdot\tan x dx = x\tan x + \log|\cos x| + C\end{aligned}$$

Example 2: Evaluate $\int x\sin 2x dx$

Solution: Integrating by parts, taking x as the first function, we get

$$\begin{aligned}\int x\sin 2x dx &= x\cdot\int \sin 2x dx - \int\left\{\frac{d}{dx}(x)\cdot\int \sin 2x dx\right\}dx \\ &= x\cdot\left(\frac{-\cos 2x}{2}\right) - \int 1\cdot\left(\frac{-\cos 2x}{2}\right)dx \\ &= \frac{-x\cos 2x}{2} + \frac{1}{2}\int \cos 2x dx \\ &= \frac{-x\cos 2x}{2} + \frac{1}{2}\cdot\frac{\sin 2x}{2} + C \\ &= \frac{-x\cos 2x}{2} + \frac{1}{4}\sin 2x + C\end{aligned}$$

Example 3: Evaluate $\int x^n\log x dx$.

Solution: Integrating by parts, taking $\log x$ as the first function, we get

$$\begin{aligned}\int x^n \log x &= (\log x)\cdot\int x^n dx - \int \left\{\frac{d}{dx}(\log x)\cdot\int x^n dx\right\}dx\\ &= (\log x)\cdot\frac{x^{n+1}}{(n+1)} - \int \frac{1}{x}\cdot\frac{x^{n+1}}{(n+1)}dx\\ &= \frac{x^{n+1}\log x}{(n+1)} - \frac{1}{(n+1)}\int x^n dx\\ &= \frac{x^{n+1}\log x}{(n+1)} - \frac{x^{n+1}}{(n+1)^2} + C.\end{aligned}$$

Example 4: Evaluate $\int x^2 \sin xdx$

Solution: Integrating by parts, taking x^2 as the first function, we get

$$\begin{aligned}\int x^2\sin xdx &= x^2\int \sin xdx - \int \left[\frac{d}{dx}(x^2)\cdot\int \sin xdx\right]dx\\ &= x^2(-\cos x) - \int 2x(-\cos x)dx\\ &= -x^2\cos x + 2\int x\cos xdx\\ &= -x^2\cos x + 2\left[x(\sin x) - \int \left\{\frac{d}{dx}(x)\cdot\int \cos xdx\right\}dx\right]\\ &= -x^2\cos x + 2\left[x\sin x - \int \sin xdx\right]\\ &= -x^2\cos x + 2[x\sin x + \cos x] + C.\end{aligned}$$

Example 5: Evaluate $\int x\cos^2 xdx$

Solution: $\int x\cos^2 xdx = \int x\left(\frac{1+\cos 2x}{2}\right)dx = \frac{1}{2}\int xdx + \frac{1}{2}\int x\cos 2xdx$

$$= \frac{x^2}{4} + \frac{1}{2}\cdot\left[x\cdot\int \cos 2xdx - \int \left\{\frac{d}{dx}(x)\cdot\int \cos 2xdx\right\}dx\right]$$

[integrating $x\cos 2x$ by parts]

$$\begin{aligned}&= \frac{x^2}{4} + \frac{1}{2}\left[\frac{x\sin 2x}{2} - \int \frac{\sin 2x}{2}dx\right]\\ &= \frac{x^2}{4} + \frac{x\sin 2x}{4} - \frac{1}{4}\cdot\frac{(-\cos 2x)}{2} + C\\ &= \frac{x^2}{4} + \frac{x\sin 2x}{4} + \frac{\cos 2x}{8} + C.\end{aligned}$$

Example 6: Evaluate $\int \log xdx$

Solution: Integrating by parts, taking $\log x$ as the first function and 1 as the second function, we get

$$\begin{aligned}\int \log xdx &= \int (\log x\cdot 1)dx\\ &= (\log x)\cdot\int 1dx - \int \left\{\frac{d}{dx}(\log x)\cdot\int 1dx\right\}dx\\ &= (\log x)\cdot x - \int \left(\frac{1}{x}\cdot x\right)dx = x\log x - \int dx\\ &= x\log x - x + C = x(\log x - 1) + C.\end{aligned}$$

Example 7: Evaluate $\int \log(1+x^2)dx$.

Solution: Integrating by parts, taking $\log(1+x^2)$ as the first function and 1 as the second function, we get

$$\begin{aligned}
\int \log(1+x^2)dx &= \int \{\log(1+x^2)\cdot 1\}dx \\
&= \log(1+x^2)\cdot\int 1dx - \int \left[\frac{d}{dx}\{\log(1+x^2)\}\cdot\int 1dx\right]dx \\
&= \log(1+x^2)\cdot x - \int \frac{2x}{(1+x^2)}\cdot x dx \\
&= x\log(1+x^2) - 2\int \frac{x^2}{(1+x^2)}dx \\
&= x\log(1+x^2) - 2\int \left(1-\frac{1}{1+x^2}\right)dx \\
&= x\log(1+x^2) - 2\int dx + 2\int \frac{dx}{(1+x^2)} \\
&= x\log(1+x^2) - 2x + 2\tan^{-1} x + \mathrm{C}.
\end{aligned}$$

Example 8: Evaluate $\int (\log x)^2 dx$.

Solution: Integrating by parts, taking $(\log x)^2$ as the first function and 1 as the second function, we get

$$\begin{aligned}
\int (\log x)^2 dx &= \int \{(\log x)^2\cdot 1\}dx \\
&= (\log x)^2\cdot\int 1dx - \int \left\{\frac{d}{dx}(\log x)^2\cdot\int 1dx\right\}dx \\
&= x(\log x)^2 - \int \left(\frac{2\log x}{x}\cdot x\right)dx \\
&= x(\log x)^2 - 2\int (\log x\cdot 1)dx \\
&= x(\log x)^2 - 2\left[(\log x)\int dx - \int \left\{\frac{d}{dx}(\log x)\cdot\int dx\right\}dx\right] \\
&= x(\log x)^2 - 2\left[x\log x - \int \frac{1}{x}\cdot x dx\right] \\
&= x(\log x)^2 - 2x\log x + 2x + C.
\end{aligned}$$

Example 9: Evaluate $\int \frac{\log x}{x^2}dx$.

Solution: Integrating by parts, taking $\log x$ as the first function and $\frac{1}{x^2}$ as the second function, we get

$$\begin{aligned}
\int \frac{\log x}{x^2}dx &= \int (\log x)\cdot\frac{1}{x^2}dx \\
&= (\log x)\cdot\int \frac{1}{x^2}dx - \int \left\{\frac{d}{dx}(\log x)\cdot\int \frac{1}{x^2}dx\right\}dx \\
&= (\log x)\left(-\frac{1}{x}\right) - \int \frac{1}{x}\cdot\left(-\frac{1}{x}\right)dx = -\frac{\log x}{x} + \int \frac{1}{x^2}dx \\
&= -\frac{\log x}{x} - \frac{1}{x} + C = \frac{-(\log x + 1)}{x} + C.
\end{aligned}$$

Example 10: Evaluate $\int e^{2x}\sin\ x dx$.

Solution: Integrating by parts, we get

$$\int e^{2x}\sin\ x dx = \left(e^{2x}\cdot\int \sin\ x dx\right) - \int \left\{\frac{d}{dx}(e^{2x})\cdot\int \sin\ x dx\right\}dx$$

$$= e^{2x}\cdot(-\cos\ x) - 2\int e^{2x}(-\cos\ x)dx$$

$$= -e^{2x}\cos\ x + 2\int e^{2x}\cos\ x dx$$

$$= -e^{2x}\cos\ x + 2\left[\left(e^{2x}\cdot\int \cos\ x dx\right) - \int \left\{\frac{d}{dx}(e^{2x})\cdot\int \cos\ x dx\right\}dx\right]$$

[integrating $e^{2x}\cos\ x$ by parts]

$$\therefore\quad 5\int e^{2x}\sin\ x dx = e^{2x}(2\sin\ x - \cos\ x) + C$$

$$\text{or}\quad \int e^{2x}\sin\ x dx = \frac{1}{5}e^{2x}(2\sin\ x - \cos\ x) + C.$$

Example 11: Evaluate $\int \left(\frac{x-\sin\ x}{1-\cos\ x}\right)dx$.

Solution: $\int \left(\frac{x-\sin\ x}{1-\cos\ x}\right)dx = \int \frac{x}{(1-\cos\ x)}dx - \int \frac{\sin\ x}{(1-\cos\ x)}dx$

$$= \int \frac{x}{2\sin^2(x/2)}dx - \int \frac{2\sin(x/2)\cos(x/2)}{2\sin^2(x/2)}dx$$

$$= \frac{1}{2}\int x\operatorname{cosec}^2(x/2)dx - \int \cot(x/2)dx$$

$$= \frac{1}{2}\left[x\cdot\int \operatorname{cosec}^2(x/2)dx - \int \left\{\frac{d}{dx}(x)\cdot\int \operatorname{cosec}^2(x/2)dx\right\}dx\right] - \int \cot\frac{x}{2}\ dx$$

$$= \left[x\cdot\left(-2\cot\frac{x}{2}\right) - \int \left[1\cdot\left(-2\cot\frac{x}{2}\right)\right]dx - \int \cot(x/2)dx + C\right.$$

$$= -x\cot\frac{x}{2} + \int \cot\frac{x}{2}dx - \int \cot\frac{x}{2}dx + C = -x\cot\frac{x}{2} + C.$$

Example 12: Evaluate $\int x\tan^{-1}\ x dx$.

Solution: Integrating by parts, taking $\tan^{-1}\ x$ as the first function, we get

$$\int x\tan^{-1}\ x\ dx = (\tan^{-1}\ x)\cdot\int x dx - \int \left\{\frac{d}{dx}(\tan^{-1}\ x)\cdot\int x dx\right\}dx$$

$$= (\tan^{-1}\ x)\cdot\frac{x^2}{2} - \int \frac{1}{(1+x^2)}\cdot\frac{x^2}{2}dx$$

$$= \frac{x^2\tan^{-1}\ x}{2} - \frac{1}{2}\int \left(1-\frac{1}{1+x^2}\right)dx\ \text{[on dividing } x^2 \text{ by } 1+x^2\text{]}$$

$$= \frac{x^2\tan^{-1}\ x}{2} - \frac{1}{2}\int dx + \frac{1}{2}\int \frac{1}{(1+x^2)}dx$$

$$= \frac{x^2\tan^{-1}\ x}{2} - \frac{x}{2} + \frac{1}{2}\tan^{-1}\ x + C$$

$$= \frac{1}{2}(1+x^2)\tan^{-1}\ x - \frac{1}{2}x + C.$$

Example 13: Evaluate $\int x^2 \sin^{-1} x dx$.

Solution: Integrating by parts, taking $\sin^{-1} x$ as the first function we get

$$\begin{aligned}
\int x^2 \sin^{-1} x dx &= (\sin^{-1} x) \cdot \frac{x^3}{3} - \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{x^3}{3} dx \\
&= \frac{x^3 \sin^{-1} x}{3} - \frac{1}{3} \int \frac{x^3}{\sqrt{1-x^2}} dx \\
&= \frac{x^3 \sin^{-1} x}{3} - \frac{1}{3} \int \frac{x \cdot x^2}{\sqrt{1-x^2}} dx \\
&= \frac{x^3 \sin^{-1} x}{3} + \frac{1}{3} \int \frac{t(1-t^2)}{t} dt, \text{ where } (1-x^2) = t^2 \\
&= \frac{x^3 \sin^{-1} x}{3} + \frac{1}{3} \int dt - \frac{1}{3} \int t^2 dt \\
&= \frac{x^3 \sin^{-1} x}{3} + \frac{1}{3} t - \frac{1}{9} t^3 + C \\
&= \frac{x^3 \sin^{-1} x}{3} + \frac{1}{3} \sqrt{1-x^2} - \frac{1}{9} (1-x^2)^{3/2} + C.
\end{aligned}$$

Example 14: Evaluate $\int \cos^{-1} x dx$

(ii) $\int \tan^{-1} x dx$

(iii) $\int \sec^{-1} x dx$

Solution: Put $\cos^{-1} x = t$ so that $x = \cos t$ and $dx = -\sin t dt$.

$$\begin{aligned}
\therefore \int \cos^{-1} x dx &= -\int t \sin t dt \\
&= -[t \cdot (-\cos t) - \int 1 \cdot (-\cos t) dt] \\
&= t \cos t - \int \cos t dt = t \cos t - \sin t + C \text{[integrating by parts]} \\
&= x \cos^{-1} x - \sqrt{1-x^2} + C \\
&\left[\because \cos t = x \Rightarrow \sin t = \sqrt{1-x^2}\right].
\end{aligned}$$

Example 15: Evaluate $\int \tan^{-1} x dx$

Solution: Put $\tan^{-1} x = t$ so that $x = \tan t$ and $dx = \sec^2 t dt$.

$$\begin{aligned}
\therefore \int \tan^{-1} x dx &= \int t \sec^2 t dt \\
&= t \cdot \tan t - \int 1 \cdot \tan t dt \quad \text{[integrating by parts]} \\
&= t \cdot \tan t + \log |\cos t| + \mathrm{C} \\
&= (\tan^{-1} x) \cdot x + \log \left| \frac{1}{\sqrt{1+x^2}} \right| + C \\
&\left[\because \tan t = x \Rightarrow \cos t = \frac{1}{\sqrt{1+x^2}}\right] \\
&= x(\tan^{-1} x) - \frac{1}{2} \log |1+x^2| + C.
\end{aligned}$$

Example 16: Evaluate $\int \sec^{-1} x dx$

Solution: Put $\sec^{-1} x = t$ so that $x = \sec t$ and $dx = \sec t \tan t dt$.

$$\therefore \int \sec^{-1} x dx = \int t(\sec t \tan t) dt$$

$$= t(\sec t) - \int 1 \cdot \sec t dt \quad \text{[integrating by parts]}$$

$$= t(\sec t) - \log |\sec t + \tan t| + C$$

$$= t(\sec t) - \log \left|\sec t + \sqrt{\sec^2 t - 1}\right| + C$$

$$= x(\sec^{-1} x) - \log \left|x + \sqrt{x^2 - 1}\right| + C.$$

Example 17: Evaluate $\int (\sin^{-1} x)^2 dx$.

Solution: Putting $x = \sin t$ and $dx = \cos t dt$, we get

$$\int (\sin^{-1} x)^2 dx = \int t^2 \cos t dt$$

$$= t^2 \cdot (\sin t) - \int 2t(\sin t) dt \quad \text{[integrating by parts]}$$

$$= t^2 \sin t - 2\left[t(-\cos t) - \int 1 \cdot (-\cos t) dt\right]$$

$$= t^2 \sin t + 2t \cos t - 2\sin t + C$$

$$= x(\sin^{-1} x)^2 + 2(\sin^{-1} x)\sqrt{1 - x^2} - 2x + C.$$

Example 18: Evaluate $\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx$.

Solution: Put $x = \sin t$ so that $dx = \cos t dt$ and $t = \sin^{-1} x$.

$$\therefore \int \frac{\sin^{-1} x}{(1 - x^2)^{3/2}} dx = \int \frac{t \cos t}{(1 - \sin^2 t)^{3/2}} dt = \int \frac{t \cos t}{\cos^3 t} dt$$

$$= \int t \sec^2 t dt$$

$$= t \cdot (\tan t) - \int 1 \cdot \tan t dt \quad \text{[integrating by parts]}$$

$$= t \cdot (\tan t) + \log |\cos t| + C$$

$$= (\sin^{-1} x) \cdot \frac{x}{\sqrt{1 - x^2}} + \log \left|\sqrt{1 - x^2}\right| + C.$$

$$\left[\because \quad \cos t = \sqrt{1 - x^2} \text{ and } \tan t = \frac{x}{\sqrt{1 - x^2}}\right]$$

$$= \frac{x(\sin^{-1} x)}{\sqrt{1 - x^2}} + \frac{1}{2} \log |(1 - x^2)| + C.$$

Example 19: Evaluate $\int \sec^3 x dx$.

Solution: $\int \sec^3 x dx = \int \sec x \cdot \sec^2 x dx$

$$= \sec x \cdot (\tan x) - \int \sec x \tan x (\tan x) dx$$

[integrating by parts]

$$= \sec x \tan x - \int \sec x (\sec^2 x - 1) dx$$

$$= \sec x \tan x - \int \sec^3 x dx + \int \sec x dx$$

$$\therefore \quad 2\int \sec^3 x dx = \sec x \tan x + \log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| + C$$

$$\text{or} \int \sec^3 x dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| + C'.$$

Example 20: Evaluate $\int \tan^{-1} \sqrt{x} dx$.

Solution: Put $\sqrt{x} = t$ so that $\frac{1}{2\sqrt{x}} dx = dt$ or $dx = 2t dt$.

$$\begin{aligned}
\therefore \int \tan^{-1} \sqrt{x} dx &= 2\int t(\tan^{-1} t) dt \\
&= 2\left[(\tan^{-1} t)\cdot\frac{t^2}{2} - \int \left\{\frac{1}{(1+t^2)}\cdot\frac{t^2}{2}\right\} dt\right] + C \\
&= t^2(\tan^{-1} t) - \int \frac{t^2}{(1+t^2)} dt + C \\
&= t^2(\tan^{-1} t) - \int \frac{[(1+t^2)-1]}{(1+t^2)} dt + C \\
&= t^2(\tan^{-1} t) - \int dt + \int \frac{1}{(1+t^2)} dt + C \\
&= t^2(\tan^{-1} t) - t + \tan^{-1} t + C = (t^2+1)\tan^{-1} t - t + C \\
&= (x+1)\tan^{-1} \sqrt{x} - \sqrt{x} + C.
\end{aligned}$$

15.4 LET US SUM UP

So, in this first unit we have learned about the

1. Concept of integration by parts and its significance in calculus.
2. Derivation of the integration by parts formula and its intuitive interpretation.
3. Technique of integration by parts through guided examples and practice problems.
4. Choosing appropriate functions to apply integration by parts effectively.
5. Integration by parts to solve a variety of integrals, including those involving logarithmic, exponential, algebraic, and trigonometric functions.

15.5 UNIT END EXERCISE

15.5.1 Evaluate $\int x^5 e^x dx$.

15.5.2 Evaluate $\int x^4 \sin x \, dx$.

15.5.3 Evaluate $\int x^n \log x \, dx$.

15.5.4 Evaluate $\int (\log x)(\log x) \, dx$.

15.5.5 Evaluate $\int e^{3x} \cos 4x \, dx$.

15.5.6 Evaluate $\int \frac{x e^x}{(x+1)^2} dx$.

15.5.7 Evaluate $\int \frac{\log x}{(1+\log x)^2} dx$.

15.5.8 Evaluate $\int \frac{e^x(1+\sin \quad)}{1+\cos x} dx$.

15.5.9 Show that $\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2}(a \sin bx - b \cos bx)$.

15.5.10 Show that $\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2+b^2}(a \cos bx + b \sin bx)$.

15.6 REFERENCES AND SUGGESTED READINGS

1. H. Anton, I. Birens and S. Davis, Calculus, John Wiley and Sons, Inc., 2002.

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3. T. M. Apostol, Calculus Vol I, Wiley & Sons (Asia) Pvt. Ltd.

4. Shanti Narayan, Integral Calculus, S. Chand, New Delhi.

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