(1) Evaluate the following integral.

$$\int x^2 \sin(2x)\ dx$$

(2) Evaluate the following integral.

$$\int_1^4 \sqrt{x} \ln x$$

(3) Evaluate the following integral.

$$\int \sin^3 x \cos^2 x \, dx$$

(4) Evaluate the following interal.

$$\int \sin^2 x \cos^2 x \, dx$$

(5) Evaluate the following integral.

$$\int \frac{1}{x\sqrt{x^2+4}} \, dx$$

(6) Evaluate the following integral:

$$\int \sqrt{x^2 - 2x} \, dx$$

(1) Evaluate the following integral.

$$\int x^2 \sin(2x)\, dx$$

$= -\frac{1}{2} x^2 \cos 2x + \frac{1}{2} x \sin x + \frac{1}{4} \cos 2x + C$

	diff	int
+	x^2	$\sin 2x$
−	$2x$	$-\frac{1}{2}\cos 2x$
+	2	$-\frac{1}{4}\sin 2x$
−	0	$\frac{1}{8}\cos 2x$

(2) Evaluate the following integral.

$$\int_1^4 \sqrt{x} \ln x$$

$\int x^{\frac{1}{2}} \ln x \, dx = \frac{2}{3} x^{3/2} \ln x - \frac{2}{3} \int x^{\frac{1}{2}} dx$

$= \frac{2}{3} x^{3/2} \ln x - \frac{4}{9} x^{3/2} + C$

	diff	int
+	$\ln x$	$x^{\frac{1}{2}}$
−	$1/x$	$\frac{2}{3} x^{3/2}$

$\int_1^4 \sqrt{x} \ln x \, dx = \frac{2}{3}(4)^{3/2} \ln 4 - \frac{4}{9}(4)^{3/2} - \frac{2}{3} 1^{3/2} \overset{0}{\ln 1} + \frac{4}{9} \cdot 1^{3/2}$

$= \boxed{\frac{16}{3} \ln x - \frac{28}{9}}$

(3) Evaluate the following integral.

$$\int \sin^3 x \cos^2 x \, dx$$

$u = \cos x \qquad -du = \sin x \, dx$

$\int \sin^3 x \cos^2 x \, dx = \int \sin^2 x \cos^2 x \sin x \, dx = \int (1 - \cos^2 x) \cos^2 x \sin x \, dx$

$= -\int (1 - u^2) u^2 du = -\frac{1}{3} u^3 + \frac{1}{5} u^5 + C = -\frac{1}{3} \cos^3 x + \frac{1}{5} \cos^5 x + C$

(4) Evaluate the following interal.

$$\int \sin^2 x \cos^2 x \, dx$$

Method #1: $\int \sin^2 x \cos^2 x \, dx = \int (1-\cos^2 x)\cos^2 x) dx = -\int \cos^4 x \, dx + \int \cos^2 x \, dx + C$

$= \frac{-1}{4} \sin x \cos^3 x - \frac{3}{4}\int \cos^2 x \, dx + \int \cos^2 x \, dx = \frac{-1}{4} \sin x \cos^3 x + \frac{1}{4}\left[\frac{1}{2}\sin x \cos x + \frac{1}{2}x\right] = -\frac{1}{4}\sin x \cos^3 x + \frac{1}{8}\sin x \cos x + \frac{1}{8}x + C$

Method #2: $\int \sin^2 x \cos^2 x \, dx = \int (\sin x \cos x)^2 dx = \int \left(\frac{1}{2}\sin 2x\right)^2 dx = \frac{1}{4}\int\left(\frac{1}{2} - \frac{1}{2}\cos 4x\right) dx = \frac{1}{8}x - \frac{1}{32}\sin 4x + C$

Method #3: $\int \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right)\left(\frac{1}{2}+\frac{1}{2}\cos 2x\right) dx = \frac{1}{4}\int (1-\cos^2 2x) dx = \frac{1}{4}\int \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx = \frac{1}{8}x - \frac{1}{32}\sin 4x + c$

Method #4: $\int \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right)\left(\frac{1}{2}+\frac{1}{2}\cos 2x\right) dx = \frac{1}{4}\int (1-\cos^2 2x) dx = \frac{1}{4}\int \sin^2 2x \, dx = \frac{1}{4}\int\left(\frac{1}{2} - \frac{1}{2}\cos 4x\right) dx = \frac{1}{8}x - \frac{1}{32}\sin 4x + c$

Method #5: $\int \sin^2\theta \cos^2\theta \, d\theta = \int \sin^2\theta (1-\sin^2\theta) d\theta = -\int \sin^4\theta \, d\theta + \int \sin^2\theta \, d\theta = \frac{-1}{4}\cos\theta \sin^3\theta + \frac{1}{4}\int \sin^2\theta \, d\theta$

$= \frac{-1}{4}\cos\theta \sin^3\theta - \frac{1}{8}\sin\theta\cos\theta + \frac{1}{8}\theta + C$

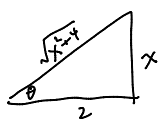
(5) Evaluate the following integral.

$$\int \frac{1}{x\sqrt{x^2+4}} \, dx$$

$x = 2\tan\theta$

$\sqrt{x^2+4} = 2\sec\theta$

$dx = 2\sec^2\theta \, d\theta$



$\int \frac{1}{x\sqrt{x^2+4}} dx = \int \frac{1}{2\sec\theta \; 2\tan\theta} \sec^2\theta \, d\theta$

$= \frac{1}{2}\int \csc\theta \, d\theta$ (I gave $\frac{12}{12}$ for getting up to here, then +3 extra credit for completing the problem)

$\alpha = \frac{\pi}{2} - \theta$

$d\alpha = -d\theta$

$= -\frac{1}{2}\int \sec\alpha \, d\alpha = -\frac{1}{2}\ln|\sec\alpha + \tan\alpha| + C$

$= -\frac{1}{2}\ln|\csc\theta + \cot\theta| + C$

$= -\frac{1}{2}\ln\left|\frac{\sqrt{x^2+4}}{x} + \frac{2}{x}\right| + C$

(6) Evaluate the following integral:

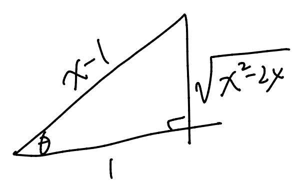
$$\int \sqrt{x^2 - 2x} \, dx$$

$\int \sqrt{x^2-2x} \, dx = \int \sqrt{(x-1)^2 - 1} \, dx$

$x - 1 = \sec\theta$

$\sqrt{x^2-2x} = \tan\theta$

$dx = \sec\theta\tan\theta \, d\theta$



$\int \sqrt{x^2-2x} \, dx = \int \tan\theta \sec\theta \tan\theta \, d\theta = \int \sec\theta (\sec^2\theta - 1) d\theta$

$= \int \sec^3\theta \, d\theta - \int \sec\theta \, d\theta = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\int \sec\theta \, d\theta - \int \sec\theta \, d\theta$

$= \frac{1}{2}\sec\theta\tan\theta - \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$

$= \frac{1}{2}(x-1)\sqrt{x^2-2x} - \frac{1}{2}\ln\left|x - 1 + \sqrt{x^2-2x}\right| + C$