(1) Use the formal limit definition of the Riemann integral to prove the following. If k is a constant and if $\int_a^b f(x)\,dx$ is Riemann integrable, then $\int_a^b kf(x)\,dx$ is Riemann integrable and

$$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$$

$$\int_a^b kf(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} kf(x_i^*)\Delta x$$

$$= \lim_{n\to\infty} k\sum_{i=1}^{n} f(x_i^*)\Delta x = k\lim_{n\to\infty} f(x_i^*)\Delta x$$

$$= k\int_a^b f(x)dx$$

(2) A bicycle travels along a road. Let

$v(t) =$ The velocity of the bicycle (in ft/sec) at time t(in sec)

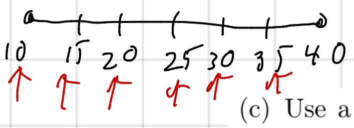
We have the following table for $v(t)$:

t	10	15	20	25	30	35	40
$v(t)$	12	10	9	8	5	3	0

(a) Write an expression representing the total displacement of the bike from 10 to 40 seconds. What are the units of your answer?

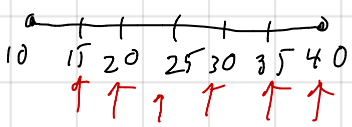
$$\boxed{\int_{10}^{40} v(t)dt \quad \text{feet}}$$

(b) Use a left hand Riemann sum with 6 subdivisions to estimate the displacement of the bike from $t = 10$ seconds to $t = 40$ seconds.



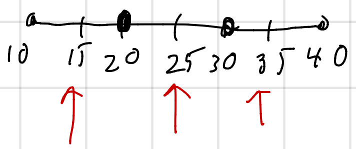
$$\boxed{(12 + 10 + 9 + 8 + 5 + 3)\cdot 5 = 235 \text{ feet}}$$

(c) Use a right hand Riemann sum with 6 subdivisions to estimate the displacement of the bike from $t = 10$ seconds to $t = 40$ seconds.



$$\boxed{(10 + 9 + 8 + 5 + 3 + 0)\cdot 5 = 175 \text{ feet}}$$

(d) Use a Riemann sum with the midpoint rule with 3 subdivisions to estimate the displacement of the bike from $t = 10$ seconds to $t = 40$ seconds.

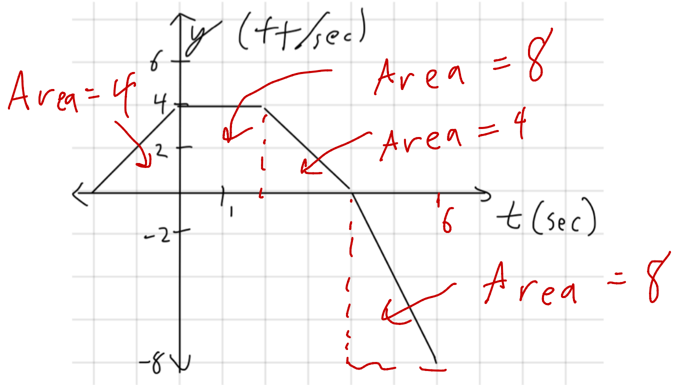


$$\boxed{(10 + 8 + 3)\cdot 10 = 210 \text{ feet}}$$

(3) A pedestrian travels along a road. Let

$$v(t) = \text{The velocity of the pedestrian (in ft/sec) at time } t \text{(in sec)}$$

Below is a graph of the velocity of the pedestrian.



(a) Write an expession for the displacement of the pedestrian from $t = -2$ seconds to $t = 6$ seconds. What are the units of your answer?

$\int_{-2}^{6} v(t)\,dt$ feet

(b) Use the graph to testimate the displacement of the pedestrian from $t = -2$ seconds to $t = 6$ seconds.

$4 + 8 + 4 - 8 = 8$ feet

(c) Write an expession for the distance the pedestrian travels from from $t = -2$ seconds to $t = 6$ seconds. What are the units of your answer?

$\int_{-2}^{6} |v(t)|\,dt$

(d) Use the graph to testimate the distance the pedestrian travels from $t = -2$ seconds to $t = 6$ seconds.

$4 + 8 + 4 + 8 = 24$ feet

(4) Maple syrup is being poured at a decreasing rate out of a tank. Let

$$r(t) = \text{The rate at which syrup is poured out of the tank (in cm}^3\text{/sec) at time } t \text{ (in sec)}$$

We have the following table of values for $r(t)$:

t	0	2	4	6	8
$r(t)$	12	10	9	8	5

(a) Write an expression for the exact amount of maple syrup that poured out of the tank between 0 and 8 seconds. What are the units of your answer?

$$\int_0^8 r(t)\,dt \quad cm^3$$

(b) Use a Riemann sum to write a lower estimate of the total amount of syrup that poured out of the tank between 0 and 8 seconds.

Since $r(t)$ is decreasing, we will use a right hand sum.

$$(10 + 9 + 8 + 5) \cdot 2 = 64 \text{ cm}^3$$

(c) Use a Riemann sum to write an upper estimate of the total amount of syrup that poured out of the tank between 0 and 8 seconds.

Since $r(t)$ is decrease, we use a left hand sum

$$(12 + 10 + 9 + 8) \cdot 2 = 78 \text{ cm}^3$$

(5) An oven is heating up. Let

$$T(t) = \text{The temperature of the oven (in } {}^\circ F\text{) at time } t \text{ (in minutes)}$$

$$f(t) = \text{The rate the oven is warming (in } {}^\circ F/\text{min) at time } t \text{ (in minutes)}$$

At time $t = 10$ minutes, the temperature is $200^\circ F$.
Answer the following problems using only the variables the problem specifies you can use. Specify the units your answers are given in. You may use integration, but not differentiation. For some of these problems, you will need to use the information that the oven is $200^\circ F$ at $t = 10$ minutes.

(a) Using $T(t)$ and no other variables, write an expression for the net increase in temperature of the oven from $t = 0$ minutes and $t = 20$ minutes.

$$T(20) - T(0) \quad {}^\circ F$$

(b) Using $f(t)$ and no other variables, write an expression for the net increase in temperature of the oven from $t = 0$ minutes and $t = 20$ minutes.

$$\int_0^{20} f(t)\,dt \quad {}^\circ F$$

(c) Using $T(t)$ and no other variables, write an expression for the temperature of the oven at $t = 20$ minutes.

$$T(20) \quad {}^\circ F$$

(d) Using $f(t)$ and no other variables, write an expression for the temperature of the oven at $t = 20$ minutes.

$$T(20) = T(10) + \int_{10}^{20} f(t)\,dt = 200 + \int_{10}^{20} f(t)\,dt \quad {}^\circ F$$

(e) Using $T(t)$ and no other variables, write an expression for the temperature of the oven at $t = 0$ minutes.

$$T(0) \quad {}^\circ F$$

(f) Using $f(t)$ and no other variables, write an expression for the temperature of the oven at $t = 0$ minutes.

$$T(0) = T(10) + \int_{10}^{0} f(t)\,dt = \boxed{200 + \int_{10}^{0} f(t)\,dt \quad {}^\circ F}$$

or

$$200 - \int_0^{10} f(t)\,dt \quad {}^\circ F$$

(6) A bike is moving along a straight path.

$$p(t) = \text{The position of the bike (in ft) at time } t \text{ (in seconds)}$$

$$v(t) = \text{velocity of the bike (in ft/sec) at time } t \text{ (in seconds)}$$

$$a(t) = \text{acceleration of the bike (in ft/sec}^2\text{) at time } t \text{ (in seconds)}$$

At time $t = 20$ seconds, the position of the bike is 500 feet, and the velocity of the bike is 20 ft/sec. Answer the following problems using only the variables the problem specifies you can use. Specify the units your answers are given in. You may use integration, but not differentiation. You may use the information the problem gives you about the position and velocity of the bike at $t = 20$ ~~minutes~~ seconds to help answer the questions.

(a) Use $v(t)$ to express the displacement of the bike from $t = 0$ to $t = 30$ seconds.

$\int_0^{30} v(t)\,dt$ feet

(b) Use $p(t)$ to express the displacement of the bike from $t = 0$ to $t = 30$ seconds.

$p(30) - p(0)$ feet

(c) Use $v(t)$ to express the distance the bike travels from $t = 0$ to $t = 30$ seconds.

$\int_0^{30} |v(t)|\,dt$ feet

(d) Use $p(t)$ to express the position of the bike at $t = 30$ seconds.

$p(30)$ feet

(e) Use $p(t)$ to express the position of the bike at $t = 0$ seconds.

$p(0)$ feet

(f) Use $v(t)$ to express the position of the bike at $t = 30$ seconds.

$500 + \int_{20}^{30} v(t)\,dt$ feet

(g) Use $v(t)$ to express the position of the bike at $t = 0$ seconds.

$500 - \int_0^{20} v(t)\,dt$ feet

-or-

$500 + \int_{20}^{0} v(t)\,dt$ feet

(h) Use $v(t)$ to express the net change in velocity from $t = 0$ to $t = 30$ seconds.

$v(30) - v(0)$ ft/sec

(i) Use $a(t)$ to express the net change in velocity from $t = 0$ to $t = 30$ seconds.

$\int_0^{30} a(t)\,dt$ ft/sec

(j) Use $v(t)$ to express the velocity at $t = 30$ seconds.

$v(30)$ ft/sec

(k) Use $a(t)$ to express the velocity at $t = 30$ seconds.

$20 + \int_{20}^{30} a(t)\,dt$ ft/sec

(l) Use $v(t)$ to express the velocity at $t = 0$ seconds.

$v(0)$ ft/sec

(m) Use $a(t)$ to express the velocity at $t = 0$ seconds.

$20 - \int_0^{20} a(t)\,dt$ ft/sec

or

$20 + \int_{20}^{0} a(t)\,dt$ ft/sec

(7) As we go deeper and deeper into a pool of water, the water pressure increases. Water pressure is measured in pounds per squaree inch. Suppose that $f(x)$ represents the instantaneous rate at which the water pressure is increasing at a depth of x feet, as measured in pounds per square inch per additional foot of depth. Suppose

$$\int_{100}^{500} f(x)\, dx = 3$$

Write a complete sentence saying what this means. Use units in your answer.

The increase in water pressure from a depth of 100 feet to a depth of 500 feet is 3 pounds per squar inch.