(b) $\int_0^{\pi^2} \frac{\sin(\sqrt{x})}{\sqrt{x}}\, dx$

$u = \sqrt{x} \quad du = \frac{1}{2\sqrt{x}}\,dx \quad \frac{1}{\sqrt{x}}\,dx = 2\,du$

Method #1

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = 2\int \sin u\,du = -2\cos u + C = -2\cos\sqrt{x} + C$$

$$\int_0^{\pi^2} \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = -2\cos\sqrt{x}\Big|_0^{\pi^2} = -2\cos\pi + 2\cos 0 = 2+2 = 4$$

Method #2

$u = \sqrt{x} \quad du = \frac{1}{2\sqrt{x}}\,dx \quad \frac{1}{\sqrt{x}}\,dx = 2\,du$

x	$u=\sqrt{x}$
0	0
π^2	π

$$\int_0^{\pi^2} \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = 2\int_0^{\pi} \sin u\,du = -2\cos u\Big|_0^{\pi}$$

$$= -2\cos\pi + 2\cos 0 = 2+2 = 4$$

(c) $\int_0^{\frac{\pi}{6}} \frac{\sin t}{\cos^2 t}\, dt$

Method #1 $\quad u = \cos t \quad du = -\sin t\,dt \quad \sin t\,dt = -du$

$$\int \frac{\sin t}{\cos^2 t}\,dt = -\int \frac{1}{u^2}\,du = \frac{1}{u} + C = \frac{1}{\cos t} + C$$

$$\int_0^{\pi/6} \frac{\sin t}{\cos^2 t}\,dt = \frac{1}{\cos t}\Big|_0^{\pi/6} = \frac{1}{\sqrt{3}/2} - \frac{1}{1} = \frac{2}{\sqrt{3}} - 1 = \frac{2\sqrt{3}}{3} - 1$$

Method #2

$u = \cos t \quad du = -\sin t\,dt \quad \sin t\,dt = -du$

$$\int_0^{\pi/6} \frac{\sin t}{\cos^2 t}\,dt = -\int_1^{\sqrt{3}/2} \frac{1}{u^2}\,du = \frac{1}{u}\Big|_1^{\sqrt{3}/2} = \frac{1}{\sqrt{3}/2} - \frac{1}{1}$$

$$= \frac{2}{\sqrt{3}} - 1 = \frac{2\sqrt{3}}{3} - 1$$

(2) Suppose $\int_0^{10} f(x)\,dx = 12$. Find $\int_0^5 f(2x)\,dx$.

x	$u=2x$
0	0
5	10

Let $u = 2x \quad du = 2\,dx \quad dx = \frac{1}{2}\,du$

$$\int_0^5 f(2x)\,dx = \frac{1}{2}\int_0^{10} f(u)\,du = \frac{1}{2}(12) = 6$$

(3) Suppose $\int_0^8 f(t)\,dt = 20$. Find $\int_0^2 t^2 f\left(t^3\right)\,dt$.

$u = t^3 \qquad du = 3t^2\,dt \qquad t^2\,dt = \frac{1}{3}\,du$

t	$u=t^3$
0	0
2	8

$$\int_0^2 t^2 f(t^3)\,dt = \frac{1}{3}\int_0^8 f(u)\,du = \frac{1}{3}(20) = \frac{20}{3}$$

(4) Suppose $\int_0^5 f(x)\,dx = 10$. Find $\int_0^{25} \frac{f(\sqrt{x})}{\sqrt{x}}dx$

x	$u=\sqrt{x}$
0	0
25	5

$u = \sqrt{x} \qquad du = \frac{1}{2\sqrt{x}}\,dx \qquad \frac{1}{\sqrt{x}}\,dx = 2\,du$

$$\int_0^{25} \frac{f(\sqrt{x})}{\sqrt{x}}\,dx = 2\int_0^5 f(u)\,du = 2\,(10) = 20$$

(5) Find $\int_0^\pi \cos x\, f(\sin x)\;dx$, where $f(x)$ is any integrable function. Justify your answer.

Let $u = \sin x \qquad du = \cos x\,dx$

x	$u=\sin x$
0	0
π	0

$$\int_0^\pi \cos x\, f(\sin x)\,dx = \int_0^0 f(u)\,du = 0$$