(1) An object with a mass of 5kg moves along a straight line. It's position along the line is given by x (in meters). An external force acts on the object, varying with position. When the object is at position x meters, the force on the object is $4\sqrt{x}$ newtons. Calculate the amount of work done by the force on the object as the object moves from $x = 1$ meter to $x = 16$ meters.

(2) Suppose that a bike moves along a straight road, and that

$$p(t) = \text{position of the bike (feet) at time } t\text{(seconds)}$$
$$v(t) = \text{velocity of the bike (feet/sec) at time } t\text{(seconds)}$$
$$a(t) = \text{acceleration of the bike (feet/sec\^{}2) at time } t\text{(seconds)}$$

(a) Write the average velocity of the bike from $t = 20$ seconds to $t = 30$ seconds in terms of $p(t)$. Include units in your answer.

(b) Write the average velocity of the bike from $t = 20$ seconds to $t = 30$ seconds in terms of $v(t)$. Include units in your answer.

(c) Write the average acceleration of the bike from $t = 20$ seconds to $t = 30$ seconds in terms of $v(t)$. Include units in your answer.

(d) Write the average acceleration of the bike from $t = 20$ seconds to $t = 30$ seconds in terms of $a(t)$. Include units in your answer.

(3) Consider the region bounded by the curves

$$y = x^2$$

$$y = x + 2$$

(a) Write an integral or integrals with respect to x representing the area of this region. You do not need to evaluate or simplify any integrals.

(b) Write an integral or integrals with respect to y representing the area of this region. You do not need to evaluate or simplify any integrals.

(c) Suppose the above region is rotated about the x-axis. Write an integral or integrals representing the volume of the resulting solid of revolution (it is your choice whether to write your integral with respect to x or to y). You do not need to evaluate or simplify any integrals.

(d) Suppose the above region is rotated about the line $x = -1$. Write an integral or integrals representing the volume of the resulting solid of revolution (it is your choice whether to write your integral with respect to x or to y). You do not need to evaluate or simplify any integrals.

(e) Suppose the above region is rotated about the line $x = 3$. Write an integral or integrals representing the volume of the resulting solid of revolution (it is your choice whether to write you integral with respect to x or to y). You do not need to evaluate or simplify any integrals.

(4) As a liquid cools, it's temperature in degrees Fahrenheit after t minutes is given by

$$f(t) = 70 + \frac{600}{10 + t}$$

(a) Find the average temperature of the liquid in degrees Fahreneheit from $t = 0$ mintues to $t = 10$ minutes.

(b) Find the average rate at which the temperature of the liquid changes in degrees Fahrenheit per minute from t =0 minutes to $t = 10$ minutes.

(5) A 10 foot chain weighing 20 pounds hangs from the top of a building.

(a) How much work must be done to pull the chain to the top of the building? Write an integral representing the work done and evaluate it.

(b) How much work must be done to pull just 5 feet of chain to the top of the building (leaving 5 feet still hanging over the side)? Write an integral representing the work done and evaluate it.

(1) An object with a mass of 5kg moves along a straight line. It's position along the line is given by x (in meters). An external force acts on the object, varying with position. When the object is at position x meters, the force on the object is $4\sqrt{x}$ newtons. Calculate the amount of work done by the force on the object as the object moves from $x = 1$ meter to $x = 16$ meters.

$$\int_1^{16} 4\sqrt{x}\,dx = \frac{8}{3}x^{3/2}\Big|_1^{16} = \boxed{168\text{ J}}$$

(2) Suppose that a bike moves along a straight road, and that

$$p(t) = \text{position of the bike (feet) at time } t\text{(seconds)}$$
$$v(t) = \text{velocity of the bike (feet/sec) at time } t\text{(seconds)}$$
$$a(t) = \text{acceleration of the bike (feet/sec\^{}2) at time } t\text{(seconds)}$$

(a) Write the average velocity of the bike from $t = 20$ seconds to $t = 30$ seconds in terms of $p(t)$. Include units in your answer.

$$\frac{p(30) - p(20)}{10} \quad \text{ft/sec}$$

(b) Write the average velocity of the bike from $t = 20$ seconds to $t = 30$ seconds in terms of $v(t)$. Include units in your answer.

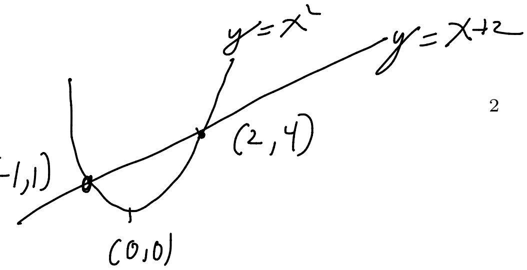
$$\frac{1}{10}\int_{20}^{30} v(t)\,dt \quad \text{ft/sec}$$

(c) Write the average acceleration of the bike from $t = 20$ seconds to $t = 30$ seconds in terms of $v(t)$. Include units in your answer.

$$\frac{v(30) - v(20)}{10} \quad \text{ft/sec}^2$$

(d) Write the average acceleration of the bike from $t = 20$ seconds to $t = 30$ seconds in terms of $a(t)$. Include units in your answer.

$$\frac{1}{10}\int_{20}^{30} a(t)\,dt \quad \text{ft/sec}^2$$



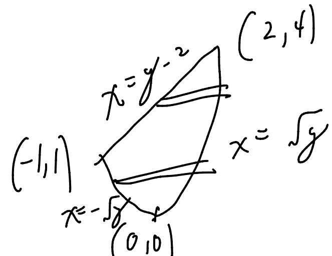
(3) Consider the region bounded by the curves

$$y = x^2$$
$$y = x + 2$$

(a) Write an integral or integrals with respect to x representing the area of this region. You do not need to evaluate or simplify any integrals.

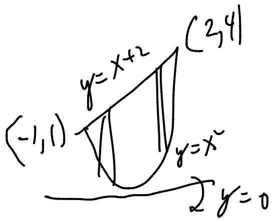
$$\int_{-1}^{2} (x+2-x^2)\,dx$$

(b) Write an integral or integrals with respect to y representing the area of this region. You do not need to evaluate or simplify any integrals.



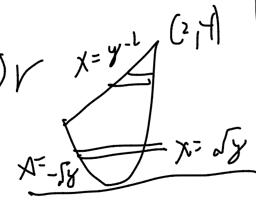
$$\int_0^1 2\sqrt{y}\,dy + \int_1^4 (\sqrt{y} - y + 2)\,dy$$

(c) Suppose the above region is rotated about the x-axis. Write an integral or integrals representing the volume of the resulting solid of revolution (it is your choice whether to write your integral with respect to x or to y). You do not need to evaluate or simplify any integrals.



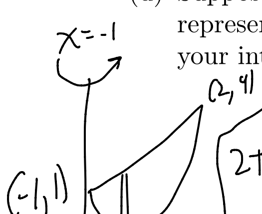
$$\boxed{\int_{-1}^{2} \left((x+2)^2 - x^4\right) dx}$$

Or



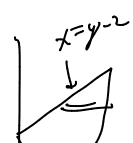
$$\boxed{2\pi \int_0^1 2\sqrt{y}\,dy + 2\pi \int_1^4 (\sqrt{y} - y + 2)\,dy}$$

(d) Suppose the above region is rotated about the line $x = -1$. Write an integral or integrals representing the volume of the resulting solid of revolution (it is your choice whether to write your integral with respect to x or to y). You do not need to evaluate or simplify any integrals.



$$\boxed{2\pi \int_{-1}^{2} (x+1)(x+2-x^2)\,dy}$$

Or

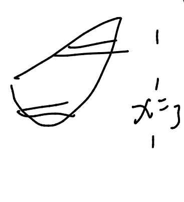


$$\boxed{\pi \int_0^1 \left((\sqrt{y}+1)^2 - (-\sqrt{y}+1)^2\right) dy + \pi \int \left((\sqrt{y}+1)^2 - (y-1)^2\right) dy}$$

(e) Suppose the above region is rotated about the line $x = 3$. Write an integral or integrals representing the volume of the resulting solid of revolution (it is your choice whether to write you integral with respect to x or to y). You do not need to evaluate or simplify any integrals.

$$\boxed{2\pi \int_{-1}^{2} (3-x)(x+2-x^2)\,dx}$$

Or



$$\boxed{\pi \int_0^1 \left[(3-(-\sqrt{y}))^2 - (3-\sqrt{y})^2\right] dy + \pi \int_1^4 \left((3-(y-2))^2 - (3-\sqrt{y})^2\right) dy}$$

(4) As a liquid cools, it's temperature in degrees Fahrenheit after t minutes is given by

$$f(t) = 70 + \frac{600}{10+t}$$

(a) Find the average temperature of the liquid in degrees Fahreneheit from $t = 0$ mintues to $t = 10$ minutes.

$$\frac{1}{10}\int_0^{10}\left[70 + \frac{600}{10+t}\right]dt = 70 + 60 \cdot \ln(10+t)\Big|_0^{10}$$

$$= 70 + 60(\ln 20 - \ln 10) = \boxed{70 + 60 \ln 2 \quad {}^{\circ}F}$$

(b) Find the average rate at which the temperature of the liquid changes in degrees Fahrenheit per minute from t =0 minutes to t = 10 minutes.

$$\frac{f(10) - f(0)}{10} = \frac{\left(70 + \frac{600}{10+10}\right) - \left(70 - \frac{600}{10+0}\right)}{10} = \frac{100 - 130}{10} = \boxed{-3 \ {}^{\circ}F/\text{min}}$$

(5) A 10 foot chain weighing 20 pounds hangs from the top of a building.

(a) How much work must be done to pull the chain to the top of the building? Write an integral representing the work done and evaluate it.

Let x = the amount of chain lifted to the top so far (ft)

$10 - x$ = remaining chain hanging over side

$f(x) = 2(10-x)$ = weight of remaining chain

$$\int_0^{10} f(x)\,dx = \int_0^{10} 2(10-x)\,dx = \left[20x - x^2\right]_0^{10} = 100 \text{ ft-lb}$$

(b) How much work must be done to pull just 5 feet of chain to the top of the building (leaving 5 feet still hanging over the side)? Write an integral representing the work done and evaluate it.

$$\int_0^5 f(x)\,dx = \left[20x - x^2\right]_0^5 = 75 \text{ ft-lb}$$