Math 1B (7:30AM) 10 March 2020

7.8 #49, 55 7.8 #13, #54 7.4 #53

8.2 #16 8.1 #13

7.8 #3 $\int_{-\infty}^{\infty} x e^{-x^2} dx \Leftarrow$ evaluate

$$= \int_{-\infty}^{0} x e^{-x^2} dx + \int_{0}^{\infty} x e^{-x^2} dx$$

evaluate both pieces

$\boxed{u = -x^2}$ $du = -2x dx$

$x dx = \frac{-1}{2} du$

$$\int_0^{\infty} x e^{-x^2} dx = \lim_{t \to \infty} \int_{x=0}^{x=t} x e^{-x^2} dx$$

$$= \lim_{t \to \infty} \frac{-1}{2} \int_{u=0}^{u=-t^2} e^{u} \, du = \lim_{t \to \infty} \left[\frac{-1}{2} e^{u} \right]_0^{-t^2} = \lim_{t \to \infty} \frac{-1}{2} e^{-t^2} + \frac{1}{2} e^{0}$$

$$= \frac{1}{2}$$

The same work shows $\int_{-\infty}^{0} x e^{-x^2} dx = -\frac{1}{2}$

$$\int_{-\infty}^{\infty} x e^{-x^2} dx = -\frac{1}{2} + \frac{1}{2} = 0$$

49–54 Use the Comparison Theorem to determine whether the integral is convergent or divergent.

49. $\int_0^\infty \frac{x}{x^3+1}\,dx$

50. $\int_1^\infty \frac{1+\sin^2 x}{\sqrt{x}}\,dx$

51. $\int_1^\infty \frac{x+1}{\sqrt{x^4-x}}\,dx$

52. $\int_0^\infty \frac{\arctan x}{2+e^x}\,dx$

53. $\int_0^1 \frac{\sec^2 x}{x\sqrt{x}}\,dx$

54. $\int_0^\pi \frac{\sin^2 x}{\sqrt{x}}\,dx$

55. The integral

$$\int_0^\infty \frac{1}{\sqrt{x}\,(1+x)}\,dx$$

49. $\int_0^\infty \frac{x}{x^3+1}\,dx$

If $x \geq 1$ the $0 < \frac{x}{x^3+1} < \frac{x}{x^3} = \frac{1}{x^2}$

$\int_1^\infty \frac{1}{x^2}\,dx$ converges

By the comparison theorem $\int_1^\infty \frac{x}{x^3+1}\,dx$ converges

$\boxed{\int_0^1 \frac{x}{x^3+1}\,dx \text{ converges}}$ $\boxed{\text{(it is a proper integral)}}$

therefore $\int_0^\infty \frac{x}{x^3+1}\,dx$ converges.

$\int_0^\infty \frac{1}{x^2}\,dx$ diverges

$\frac{x}{x^3+1} < \frac{1}{x^2}$ if $x > 0$

Theorem does not apply

55. The integral

$$\int_0^\infty \frac{1}{\sqrt{x}\,(1+x)}\,dx$$

$u = \sqrt{x}$

$$\int_0^1 \frac{1}{\sqrt{x}\,(1+x)}\,dx + \int_1^\infty \frac{1}{\sqrt{x}\,(1+x)}\,dx$$

$$= \lim_{t\to 0^+} \int_t^1 \frac{1}{\sqrt{x}\,(1+x)}\,dx + \lim_{s\to\infty} \int_1^s \frac{1}{\sqrt{x}\,(1+x)}\,dx$$

$u = \sqrt{x} \qquad u^2 = x \qquad dx = 2u\,du$

$$= \lim_{t\to 0^+} \int_{\sqrt{t}}^{\sqrt{1}} \frac{2\cancel{u}}{\cancel{u}(1+u^2)}\,du + \lim_{s\to\infty} \int_{\sqrt{1}}^{\sqrt{s}} \frac{2\cancel{u}}{\cancel{u}(1+u^2)}\,du$$

$$\lim_{t\to 0^+} 2\arctan u \Big|_{\sqrt{t}}^{\sqrt{1}} + \lim_{s\to\infty} 2\arctan u \Big|_{\sqrt{1}}^{\sqrt{s}}$$

$$= 2\cdot\frac{\pi}{4} + 2\cdot\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \pi$$

53–54 Use integration by parts, together with the techniques of this section, to evaluate the integral.

53. $\int \ln(x^2 - x + 2)\, dx$ **54.** $\int x \tan^{-1}x\, dx$

$$\int \ln(x^2-x+2)\,dx$$

	dif	int
+	$\ln(x^2-x+2)$	1
−	$\frac{2x-1}{x^2-x-2}$	x

$$= x\ln(x^2-x-2) - \int \frac{2x^2-x}{x^2-x+2}\,dx$$

~~as~~ use long division

$$\frac{2x^2-x}{x^2-x+2} = 2 + \frac{x-4}{x^2-x+2}$$

$$\frac{x-4}{\left(x-\frac{1}{2}\right)^2+\frac{9}{4}}$$

complete the squar

13. $x = \frac{1}{3}\sqrt{y}\,(y-3), \quad 1 \leqslant y \leqslant 9$

arc length for $y=1$ to $y=9$

$$\int_{y=1}^{y=9} ds = \int_1^9 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$= \int_1^9 \sqrt{1+\tfrac{1}{4}y-\tfrac{1}{2}+\tfrac{1}{4}y^{-1}}\,dy$$

$$x = \frac{1}{3}y^{3/2} - y^{1/2}$$

$$\frac{dx}{dy} = \frac{1}{2}y^{1/2} - \frac{1}{2}y^{-1/2}$$

$(a+b)^2 = a^2+2ab+b^2$

$$\left(\frac{dx}{dy}\right)^2 = \frac{1}{4}y - \frac{1}{2}\cdot 1 + \frac{1}{4}y^{-1}$$

$y^{1/2} \cdot y^{-1/2}$

$$= \int_1^9 \sqrt{\frac{1}{4}y+\frac{1}{2}+\frac{1}{4}y^{-1}}\,dy$$

$$\int_1^9 \sqrt{\left(\frac{1}{2}y^{1/2}+\frac{1}{2}y^{-1/2}\right)^2}\,dy$$

$$\int_1^9 \left(\frac{1}{2}y^{1/2}+\frac{1}{2}y^{-1/2}\right)dy = \left[\frac{1}{3}y^{3/2} + y^{1/2}\right]_1^9$$

$$= \underbrace{9+3}_{12} - \left(\frac{1}{3}+1\right) =$$

$$12 - \frac{4}{3} = \frac{32}{3}$$

Find the arclength function s for the curve $x = \sqrt{y}\,(y-3)$ with starting point $y=1$

Find the arclength function s for the curve $x = \sqrt{y}\,(y-3)$ with starting point $y=1$

$$s(y) = \int_1^y ds = \ldots\ldots = \left[\frac{1}{3}y^{3/2} + y^{1/2}\right]_1^y$$

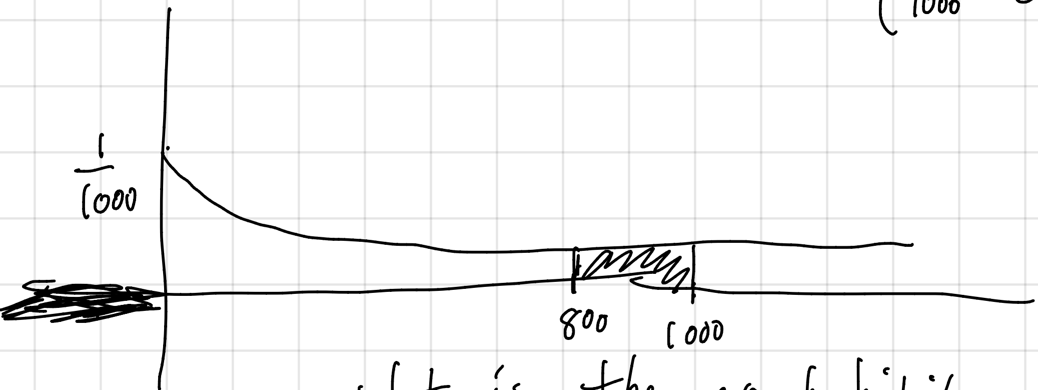
$$= \frac{1}{3}y^{3/2} + y^{1/2} - \underbrace{\frac{4}{3}}_{\frac{1}{3}\cdot 1+1}$$

I turn on a light bulb and wait for it to fail

$X =$ Time it takes (hours) for the light bulb to fail

PDF for X

$$f(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{1000} e^{\frac{-x}{1000}}, & x \geq 0 \end{cases}$$



what is the probability the light bulb lasts between 800 and 1000 hours?

$$P(800 \leq X \leq 1000) = \int_{800}^{1000} f(x)\,dx$$

$$= \int_{800}^{1000} \frac{1}{1000} e^{\frac{-x}{1000}}\,dx = \frac{-1000}{1000} e^{\frac{-x}{1000}} \Big|_{800}^{1000}$$

$$= -e^{-1} + e^{-\frac{800}{1000}} = -e^{-1} + e^{-.8}$$

$$= e^{-.8} - e^{-1}$$

$$CDF = F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx$$

$$= \underbrace{\cancel{\int_{-\infty}^{0} f(x)\,dx}}_{0} + \int_0^x f(x)\,dx = \int_0^x \frac{1}{1000} e^{\frac{-x}{1000}}\,dx$$

$$= \frac{-1000}{1000} e^{\frac{-x}{1000}} \Big|_0^x = -e^{\frac{-x}{1000}} + \underbrace{1}_{e^0}$$

$$\boxed{F(x) = 1 - e^{\frac{-x}{1000}}}$$

$$F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{\frac{-x}{1000}}, & x \geq 0 \end{cases}$$

$$F(x) = 1 - e^{\frac{-x}{1000}}$$

what is the probability the light bulb lasts between 800 and 1000 hours?

$$P(800 \le X \le 1000) = F(1000) - F(800)$$

$$= \left(1 - e^{\frac{-1000}{1000}}\right) - \left(1 - e^{\frac{-800}{1000}}\right)$$

$$= e^{-.8} - e^{-1}$$

$a < b$

$$P(a \le X \le b) = \int_a^b f(x)\,dx$$