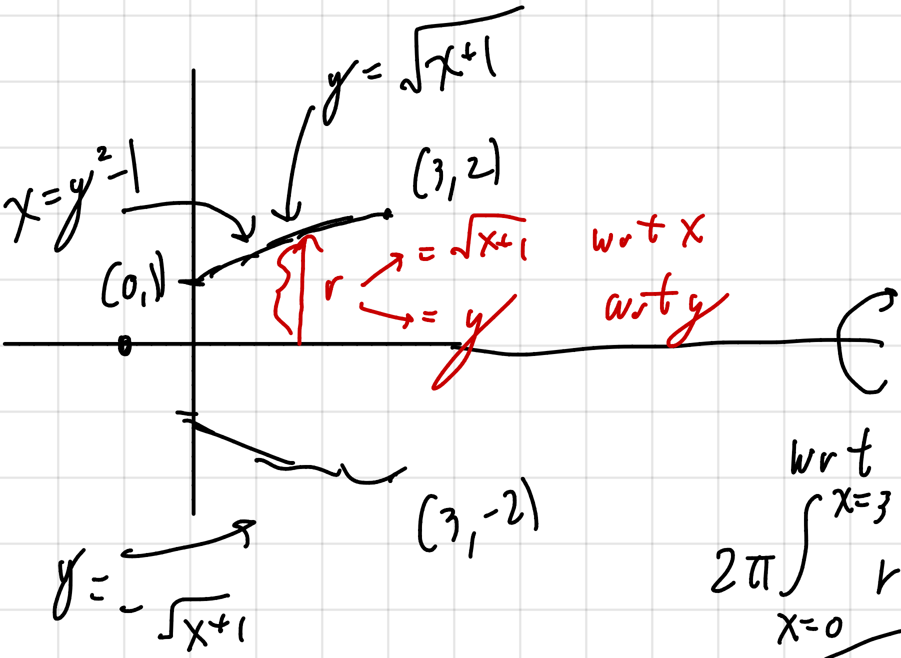
Math 1b (7:30AM) 11 March 2020

8.2 #9 wrt y 8.1 #36a

9. $y^2 = x + 1,\ 0 \leqslant x \leqslant 3$

$x = y^2 - 1$



$$\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}}$$

wrt x

$$2\pi \int_{x=0}^{x=3} r\, ds \qquad \left(\frac{dy}{dx}\right)^2$$

$$2\pi \int_0^3 \sqrt{x+1}\, \sqrt{1 + \frac{1}{4(x+1)}}\, dx$$

7–14 Find the exact area of the surface obtained by rotating the curve about the x-axis.

$$2\pi \int_{y=1}^{2} r\, ds = 2\pi \int_1^2 y \sqrt{1 + (2y)^2}\, dy \qquad \left(\frac{dx}{dy}\right)^2$$

$x = y^2 - 1$

$$\frac{dx}{dy} = 2y$$

$u = 1 + 4y^2$

$du = 8y\, dy$

$y\,dy = \frac{1}{8} du$

$$= 2\pi \int_0^3 \sqrt{x+1}\, \sqrt{1 + \frac{1}{4(x+1)}}\, dx = 2\pi \int_0^3 \sqrt{x + 1 + \frac{1}{4}}\, dx = 2\pi \int_0^3 \sqrt{x + \frac{5}{4}}\, d$$

8.1 #36a

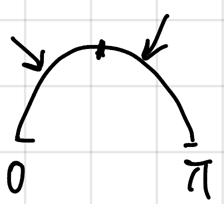
$s(y) = $ blah y blah

$x = g(y)$

$s(x) = $ blah blah

$y = f(x)$

36. (a) Find the arc length function for the curve $y = \ln(\sin x)$, $0 < x < \pi$, with starting point $(\pi/2, 0)$.

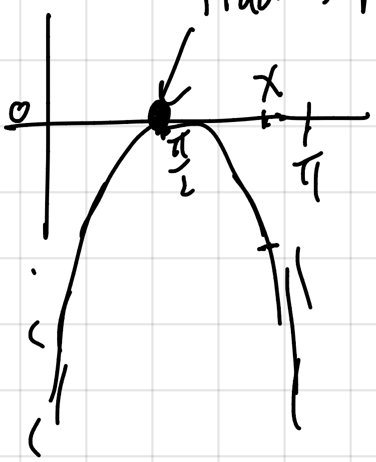


$y = \ln \sin x$

$\frac{dy}{dx} = \frac{\cos x}{\sin x} = \cot x$

$$\underline{s(x)} = \int_{x=\frac{\pi}{2}}^{x=x} ds$$

$$= \int_{\frac{\pi}{2}}^{x} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$



$$= \int_{\frac{\pi}{2}}^{x} \sqrt{1 + \cot^2 x}\, dx$$

$$= \int_{\frac{\pi}{2}}^{x} \csc x \, dx$$

$\int_{\frac{\pi}{2}}^{x} \csc t \, dt$

memorize

$$= \ln|\csc x - \cot x| \Big|_{\pi/2}^{x}$$

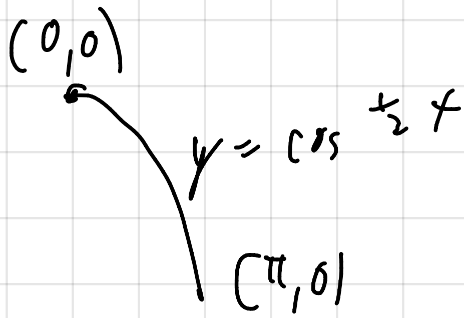
$$s(x) = \ln|\csc x - \cot x| - \underbrace{\ln\left|\csc\tfrac{\pi}{2} - \cot\tfrac{\pi}{2}\right|}_{=0}$$

$$s(x) = \ln|\csc x - \cot x|$$

8.2

11. $y = \cos\left(\frac{1}{2}x\right),\ 0 \leqslant x \leqslant \pi$

area of surface of revolution about x-axis



$\frac{dy}{dx} = -\frac{1}{2}\sin\frac{1}{2}x$

$\left(\frac{dy}{dx}\right)^2$

$$2\pi\int_0^\pi r\,ds = 2\pi\int_0^\pi \cos\tfrac{1}{2}x\sqrt{1+\left(-\tfrac{1}{2}\sin\tfrac{1}{2}x\right)^2}\,dx$$

$u = -\frac{1}{2}\sin\frac{1}{2}x \quad du = -\frac{1}{4}\cos\frac{1}{2}x\,dx$

$=$ blah blah blah $\sqrt{1+u^2}\,du$

$u = \tan\theta$

We light a light bulb

$X =$ Time until light bulb fails (hours)

PDF of X

f

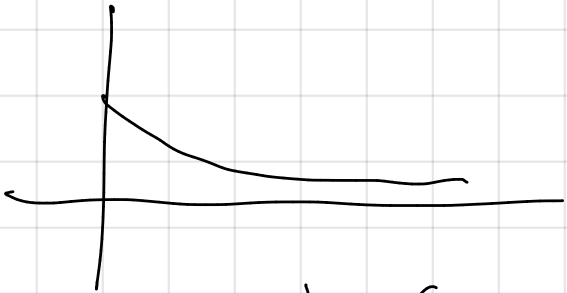
CDF of X

$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx$

$$f(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{1000} e^{\frac{-x}{1000}}, & x \ge 0 \end{cases}$$

$$F(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{\frac{-x}{1000}} & x \ge 0 \end{cases}$$

PDF of X



$F'(x) = f(x)$ (when $f(x)$ is continuous)

$$F'(x) = f(x)$$

If X is a continuous random variable, then the average value of X (also called the expected value of X or the mean value of X) is

$$\int_{-\infty}^{\infty} x f(x)\,dx \quad \text{(where } f \text{ is the PDF of } X\text{)}$$

On average, how long does a light bulb last?

If X is a continuous random variable, then the average value of X (also called the expected value of X or the mean value of X) is

$$\int_{-\infty}^{\infty} x f(x)\,dx \quad \text{(where } f \text{ is the PDF of } X\text{)}$$

On average, how long does a light bulb last?

Since X represents the lifetime of a light bulb, we are asking for the expected value of X.

$$f(x) = \begin{cases} 0, & x<0 \\ \frac{1}{1000} e^{\frac{-x}{1000}}, & x \geq 0 \end{cases}$$

$$\text{Expected value of } X = \int_{-\infty}^{\infty} x f(x)\,dx = \cancel{\int_{-\infty}^{0} x \cdot 0\,dx}^{\,0} + \int_0^{\infty} x \frac{1}{1000} e^{\frac{-x}{1000}}\,dx$$

$$= \lim_{t\to\infty} \frac{1}{1000} \int_0^t x e^{\frac{-x}{1000}}\,dx$$

	diff	int
+	x	$e^{-x/1000}$
−	1	$-1000\, e^{-x/1000}$
+	0	$1000^2\, e^{-x/1000}$

$$= \lim_{t\to\infty} \frac{1}{1000} \left[-1000 x e^{\frac{-x}{1000}} - 1000^2 e^{\frac{-x}{1000}} \right]_0^t$$

$$\lim_{t\to\infty} \left[\underbrace{-te^{\frac{-t}{1000}}}_{0} - \underbrace{1000 e^{\frac{-t}{1000}}}_{0} + 0 + 1000 \right] = 1000$$

(L'Hospital's Rule)

On average our light bulbs last 1000 hours.

$$\lim_{t\to\infty} -t e^{-t/1000} = \lim_{t\to\infty} \frac{-t}{e^{t/1000}} = \lim_{t\to\infty} \frac{-1}{\frac{1}{1000} e^{t/1000}} = 0$$