Math 1B (7:30 AM) 13 March 2020

Read 9.1 - 9.3

Differential equations

What is a differential equation?

An algebraic equation

variable x represents a number

unknown number $\rightarrow$ $x^2 + 6 = 31$

a solution to the algebraic equation is a number we can assign to x that makes the equation true

$x = 5$ $\quad 5^2 + 6 = 31$ ✓

$x = -5$ $\quad (-5)^2 + 6 = 31$ ✓

In a differential equation we have a variable, x (sometimes t) that represents an independent variable, and another variable y that represents an unknown function of x.

Our equation can include x, y, $y' = \frac{dy}{dx}$, $y'' = \frac{d^2y}{dx^2}$, ...

$y' = 2x$

A solution to the equation is a function $f(x)$ such that when we substitute $y = f(x)$ it makes the equation true.

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solutions $y = x^2$ $\qquad \frac{d}{dx}(x^2) = 2x$ ✓

$y = x^2 + 17$ $\qquad \frac{d}{dx}(x^2+17) = (x^2+17)' = 2x$ ✓

General solution: $y = x^2 + c$
(c any constant)

$$y' = f(x) \overset{\text{solution}}{\Longrightarrow} y = \int f(x)\,dx + C$$

$y = -x-1$
$\frac{dy}{dx} = -1$

$$\boxed{y' = x + y}$$

solution $y = -x-1$

$(-x-1)' = x + (-x-1)$

$-1 = x - 1 - 1$

$-1 = -1$ ✓

Given a differential equation

$y' = x + y$

$y' = 2x$ $\quad y|_{x=0} = 3$

General solution $y(0)=3$

$y = x^2 + C$

any constant

$y = x^2 + 3$

Algebraic equation

$x^2 + 6 = 31$

$x = 5$ is a solution

$5^2 + 6 = 31$ ✓

$x = -5$ is a solution

$\to (-5)^2 + 6 = 31$

$y = -x - 1$ is a solution because

$(-x-1)' = x + (-x-1)$

$-1 = -1$ ✓

$y' = x + y$

General solution

$$\boxed{y = -x - 1 + ce^{+x}}$$

check

equation $y' = x + y$

check: $(-x-1+ce^{x})' = \cancel{x+} (\cancel{-x}-1+ce^{x})$

$-1 + ce^{x} = -1 + ce^{x}$ ✓

Differential equation

$$y' = y$$

A solution

$$y = e^x$$

General solution arbitrary

$$y = Ce^x$$

$$\frac{d}{dx} Ce^x = Ce^x \checkmark$$

$$y' = y$$