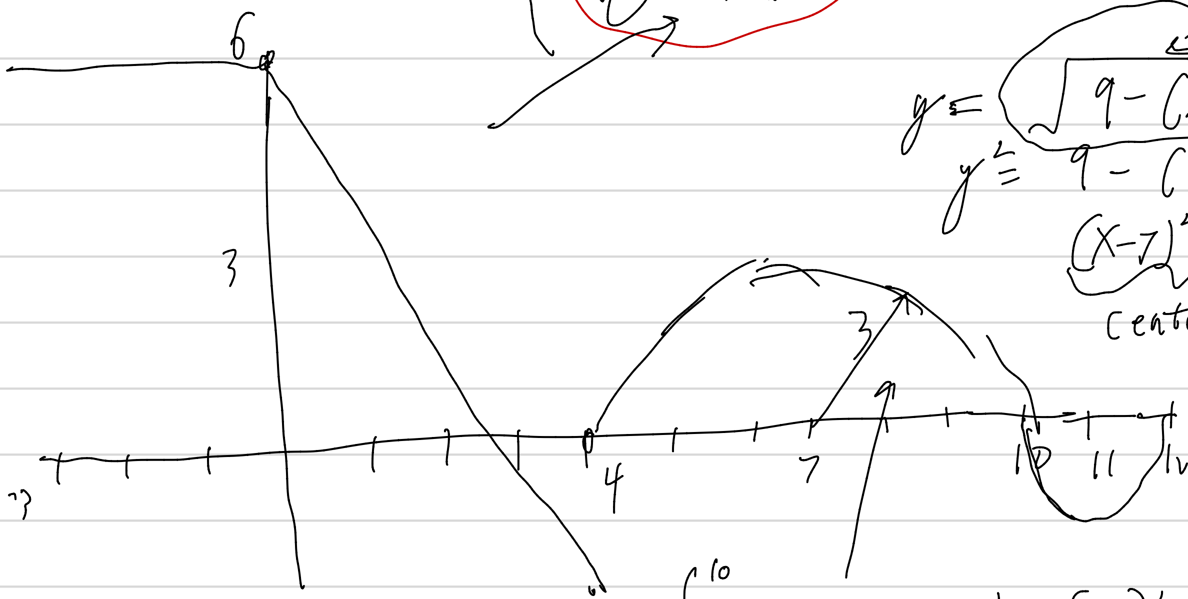
Math 1b (8:30AM) 8 Jan 2020

HW #1

$$f(x) = \begin{cases} 6 & -3 \le x < 0 \\ 6-2x & 0 \le x < 4 \\ \sqrt{9-(x-7)^2} & 4 \le x < 10 \\ -\sqrt{1-(x-11)^2} & 10 \le x \le 12 \end{cases}$$



$$y = \sqrt{9-(x-7)^2}$$

$$y^2 = 9-(x-7)^2$$

$$(x-7)^2+y^2 = 9$$

center $(7,0)$

radius 3

$$\int^{10} f(x)\,dx = \text{Area} = \frac{1}{2}\pi(3)^2$$

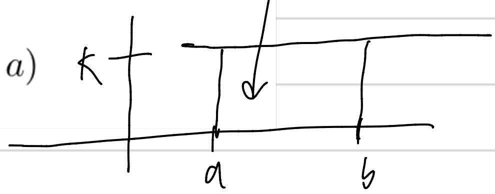
$$\int_{10}^{12} f(x)\,dx = -\left(\frac{1}{2}\pi\cdot 1^2\right)$$

$$\int_{10}^{12} |f(x)|\,dx = +\frac{1}{2}\pi$$

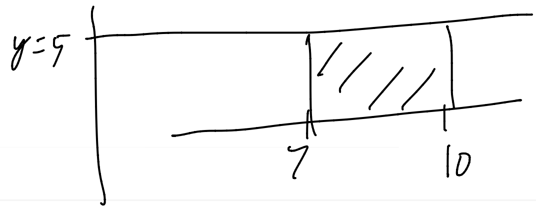
Area = height · width
= k (b − a)

Property #6 of the definite integral: If k is a constant then

$$\int_a^b k\,dx = k\,(b-a)$$



Example: Find

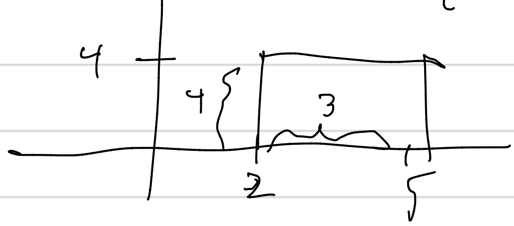


$$\int_7^{10} 5\,dx = 5\cdot(10-7)$$
$$= 5\cdot 3 = 15$$

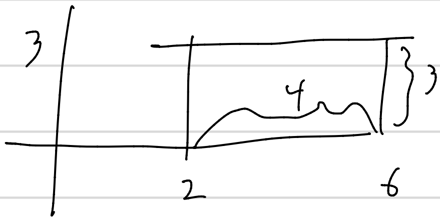
Find

$$\int_5^2 4\,dx = 4\cdot(2-5)$$
$$= -12$$

$$\int_2^5 4\,dx = 4\cdot(5-2) = 12$$

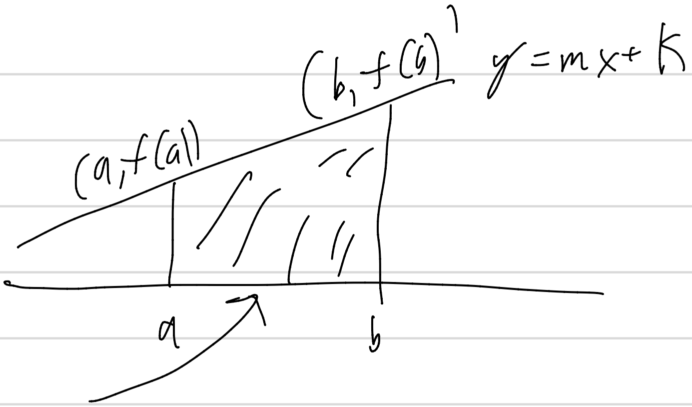


$$\int_2^6 3\,dx = 4\cdot 3 = 12$$



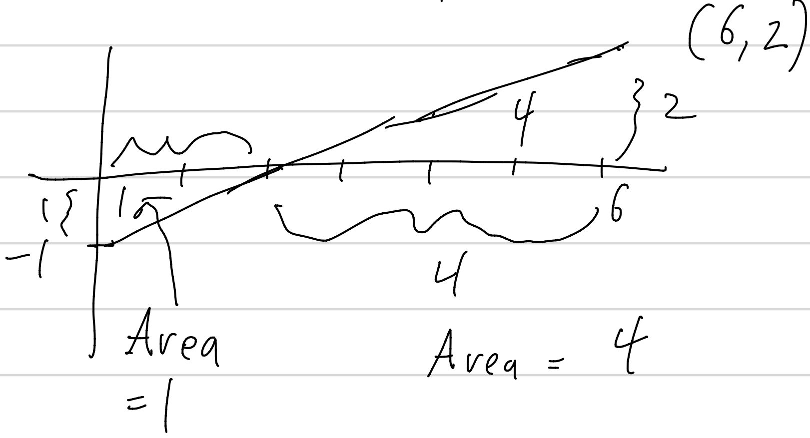
Property 7 of the definite integral. If $f(x) = mx + k$ is a linear function, then

$$\int_a^b f(x)\,dx = \frac{f(a)+f(b)}{2}(b-a)$$



Area = Average height of two sides × width = $\frac{f(a)+f(b)}{2}(b-a)$

$$\int_0^6 \left(\frac{1}{2}x - 1\right) dx = -1 + 4 = 3$$



$$f(x) = \frac{1}{2}x - 1$$

$$\int_0^6 f(x)\,dx = \frac{f(0)+f(6)}{2}\cdot(6-0)$$

$$= \frac{-1+2}{2}(6) = 3$$

Property 8 of the definite integral.

$$\int_a^b (f(x) + g(x))\, dx = \int_a^b f(x)\, dx + \int_a^b g(x)\, dx$$

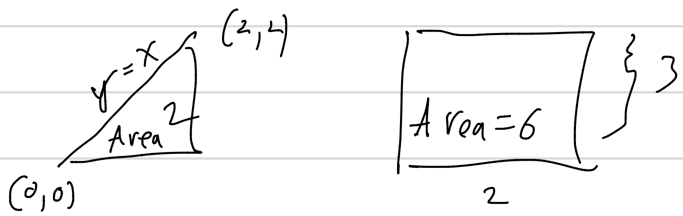
$$\int_a^b (f(x) - g(x))\, dx = \int_a^b f(x)\, dx - \int_a^b g(x)\, dx$$

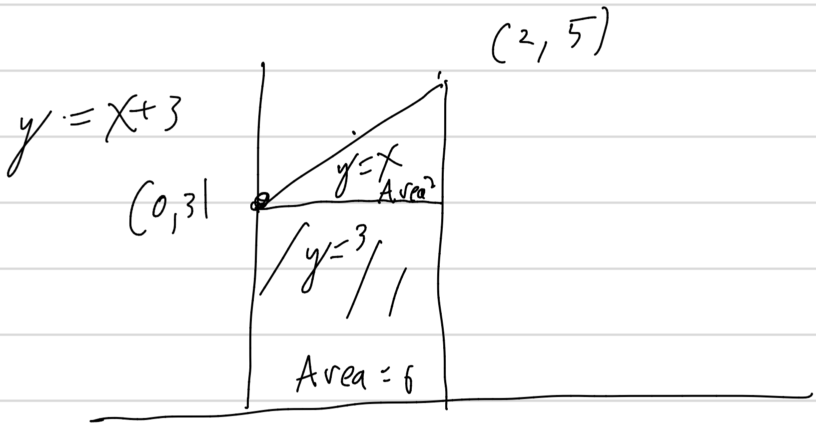
The integral of a sum is the sum of the integrals.
The integral of a difference is the difference of the integrals.

Example:

$$\int_0^2 (x+3)\, dx$$

$$\int_0^2 (x+3)\, dx = \int_0^2 x\, dx + \int_0^2 3\, dx = 2 + 6 = 8$$





Property 9 of the definite integral.

$$\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$$

We say that the variable of integration x is a "dummy variable."

Another example of dummy variable, is in summations

$$\sum_{i=1}^{5} i^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

$$\sum_{k=1}^{5} k^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

i and k are dummy variable,

$$\sum_{i=1}^{5} i^2 = \sum_{k=1}^{5} k^2$$

If I let k = 42, then $\sum_{k=1}^{5} k^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$

Property 10 of the definite integral: If $f(x) = g(x)$ for all x in an interval $[a, b]$, except perhaps for a finite number of points, then

$$\int_a^b f(x)\,dx = \int_a^b g(x)\,dx$$

In other words, if we modify a function at a finite number of points in an interval, it does not change the value of the integral.

Example: Suppose

$$f(x) = \begin{cases} 3 & x < 5 \\ 7 & x \geq 5 \end{cases}$$

Find

$$\int_0^5 f(x)\,dx$$

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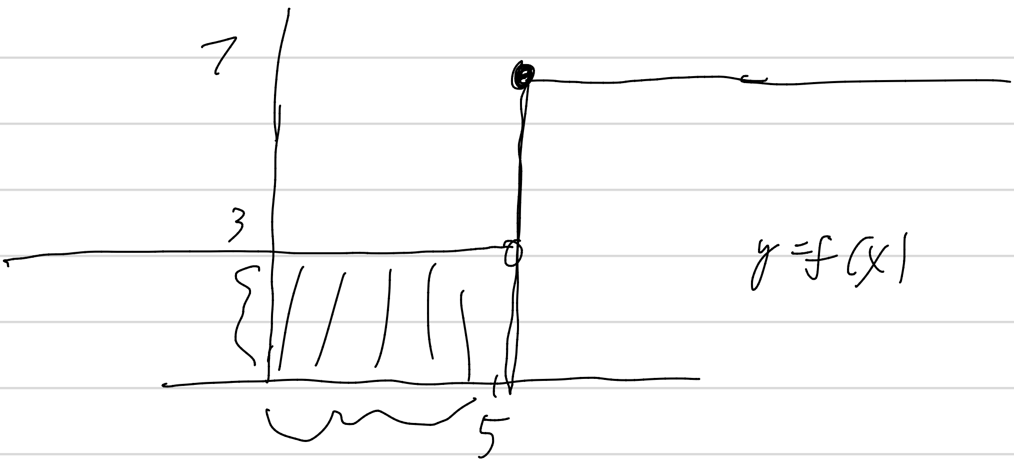
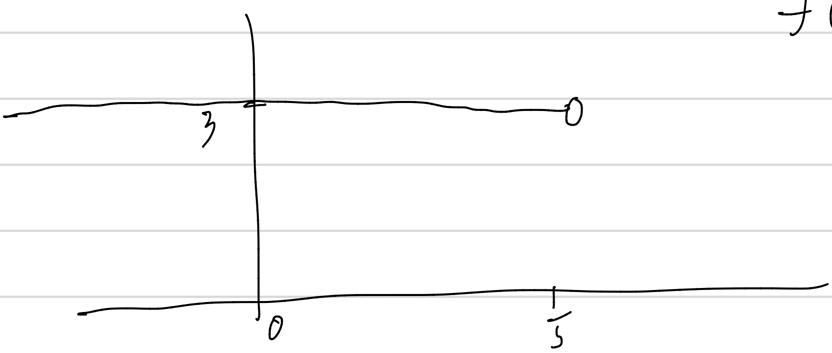


$$\int_0^5 f(x)\, dx = 3(5-0) = 15$$



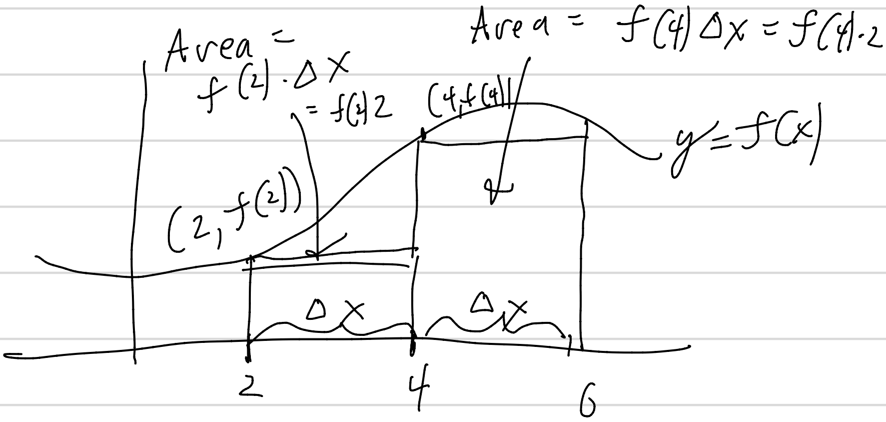
$f(x) = 3$ for $x < 5$ and is undefined otherwise

$f(5)$ is undefined

$$\int_0^5 f(x)\, dx = 15$$

Riemann Sums

$$\int_2^6 f(x)\,dx$$

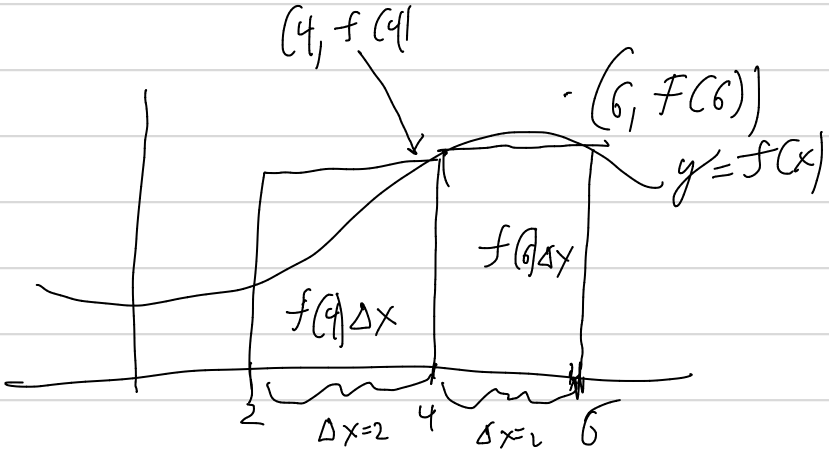


Riemann Sum n = 2 subdivisions
Left hand sum (LHS)

$$\Delta x = \frac{6-2}{2} = \frac{4}{2} = 2$$

$$f(2)\cdot \overset{\Delta x}{2} + f(4)\cdot 2 = (f(2)+f(4))\cdot 2$$

$$\int_2^6 f(x)\,dx$$

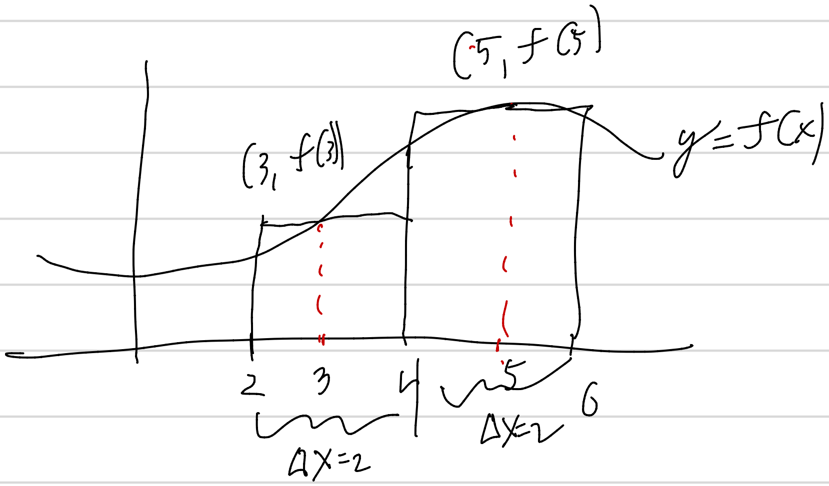


Riemann Sum n=2 subdivisions
RHS
Right hand sum

$$\Delta x = \frac{6-2}{2} = 2$$

$$(f(4) + f(6))\,\overset{\Delta x}{2}$$

$$\int_2^6 f(x)\,dx$$



Riemann Sum, using the midpoint rule

$n = 2$

$$(f(3) + f(5))\,2$$