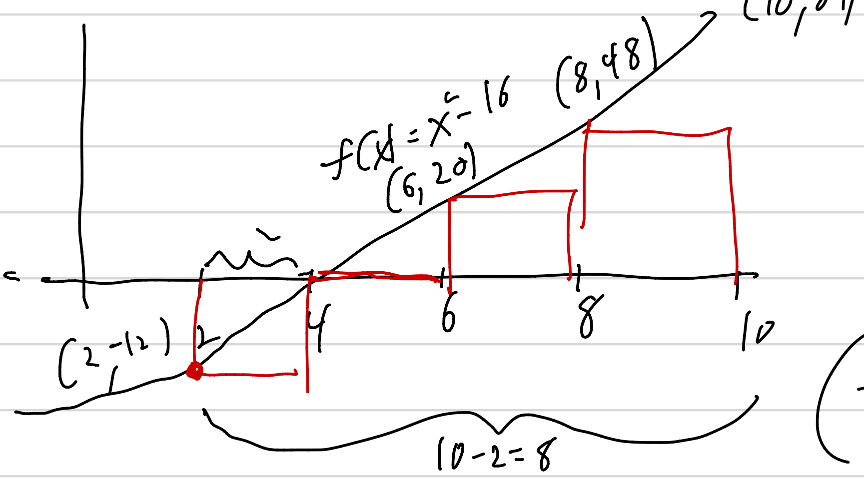
Math 16 (8:30AM) 9 Jan 2020

$$\int_2^{10} (x^2-16)\,dx$$

Write a Riemann sum with 4 subdivisions using left end points for this integral.



$$\Delta x = \frac{10-2}{4} = 2$$

$$\left(f(2)+f(4)+f(6)+f(8)\right)\Delta x$$

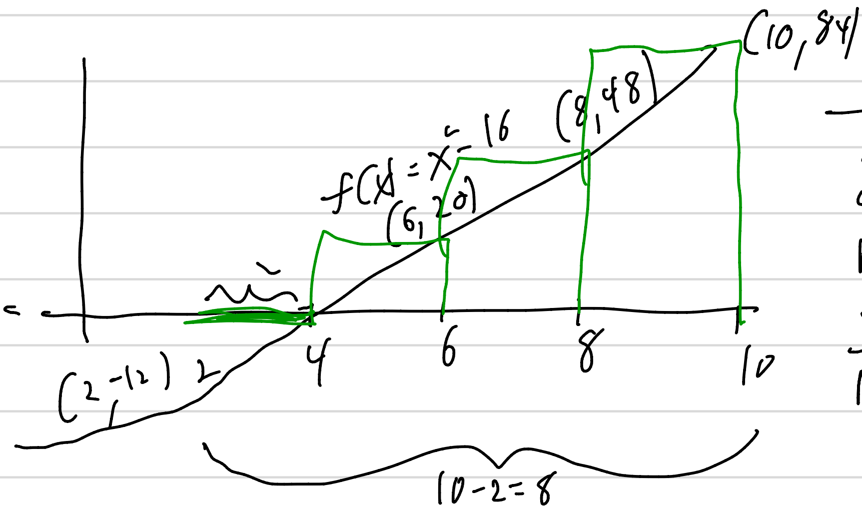
$$(-12+0+20+48)\cdot 2$$

$$=(56)2 = 112$$

LHS:

If $f(x)$ is increasing then the LHS will underestimate the true value of $\int_a^b f(x)\,dx$

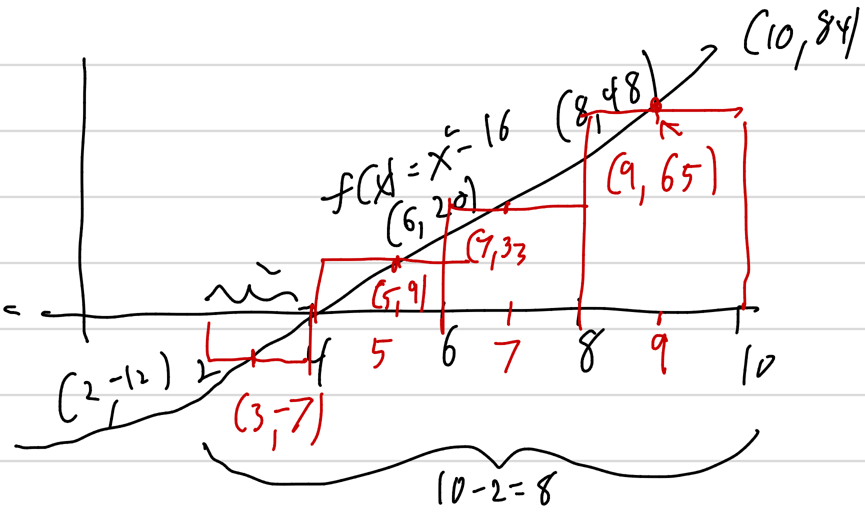
If $f(x)$ is decreasing, LHS will overestimate $\int_a^b f(x)\,dx$



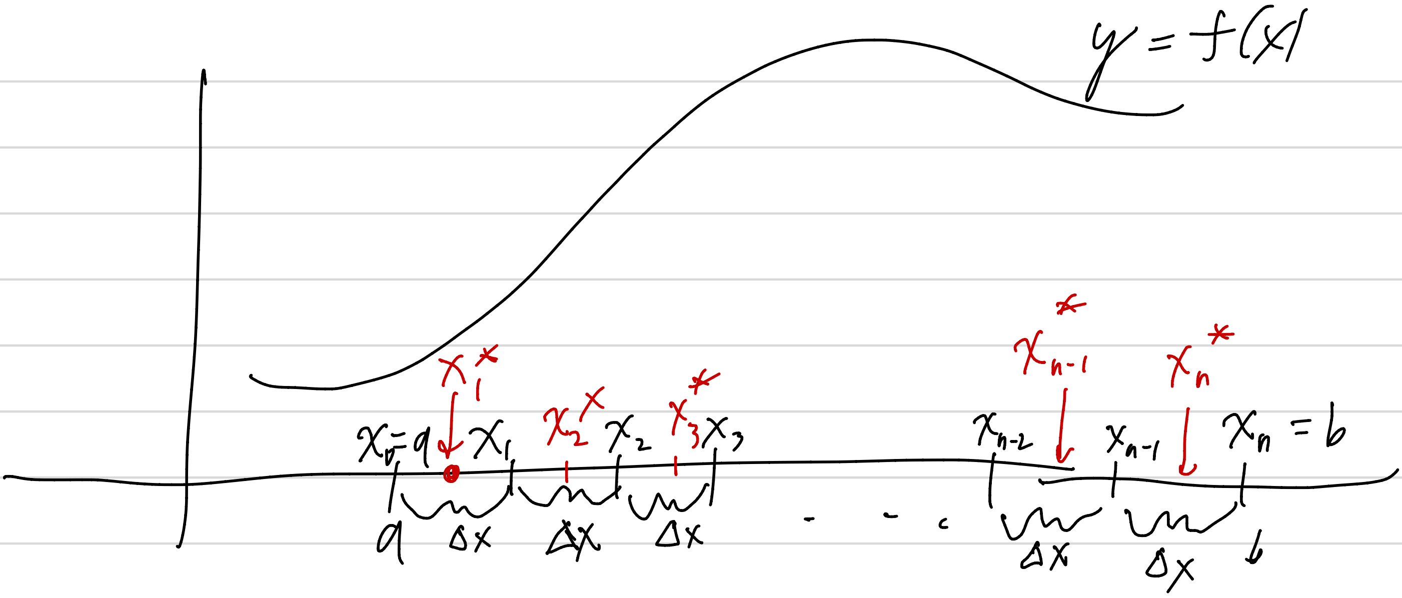
If $f(x)$ is increasing, the RHS will overestimate $\int_a^b f(x)\,dx$

If $f(x)$ is decreasing, the RHS will underestimate $\int_a^b f(x)\,dx$

$$\text{RHS} = (0+20+48+84)\,2 = 304$$



Choosing midpoints: $(-7 + 9 + 33 + 65) \cdot 2$



Write the Riemann sum with n subdivisions for

$$\int_a^b f(x)\,dx$$

$$\Delta x = \frac{b-a}{n}$$

$x_0 = a \quad x_1 = a + \Delta x \quad x_2 = a + 2\Delta x, \ldots x_k = a + k\Delta x$

$$x_k = a + k\Delta x = a + \frac{k(b-a)}{n}$$

Choose a sample point from each subinterval, x_k^*

$x_{k-1} \le x_k^* \le x_k \qquad$ LHS: $x_k^* = x_{k-1}$, RHS: $x_k^* = x_k$

Midpoint: $x_k^* = \dfrac{x_{k-1} + x_k}{2}$

Our Riemann sum is $\displaystyle\sum_{k=1}^{n} f(x_k^*)\Delta x$

$$= \left(\sum_{k=1}^{n} f(x_k)^*\right)\Delta x$$

$$\sum_{k=1}^{n} f(x_k^*)\Delta x = f(x_1^*)\Delta x + f(x_2^*)\Delta x + \ldots f(x_n^*)\Delta x$$

Choose a sample point from each subinterval, x_k^*,

$x_{k-1} \le x_k^* \le x_k$ LHS: $x_k^* = x_{k-1}$, RHS: $x_k^* = x_k$

Midpoint: $x_k^* = \frac{x_{k-1} + x_k}{2}$

Our Riemann sum is $\sum_{k=1}^{n} f(x_k^*) \Delta x$

$= \left(\sum_{k=1}^{n} f(x_k)^* \right) \Delta x$

$$\sum_{k=1}^{n} f(x_k^*) \Delta x = f(x_1^*) \Delta x + f(x_2^*) \Delta x + \ldots f(x_n^*) \Delta x$$

Definition of the definite integral

$$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k^*) \Delta x$$

In order for $\int_a^b f(x)\,dx$ to exist,

$\lim_{n \to \infty} \sum_{k=1}^{n} f(x_k^*) \Delta x$ must exist, and

the limit must be the same no matter how we choose the x_k^*.

If $\int_a^b f(x)\,dx$ exists, we say $\int_a^b f(x)\,dx$ is "integrable"

Otherwise it is not integrable.

Theorem

If f is continuous on $[a,b]$

then $\int_a^b f(x)\,dx$ exists.