Math 1b (8:30AM) 10 Jan 2020

Take home Quiz - due Tuesday

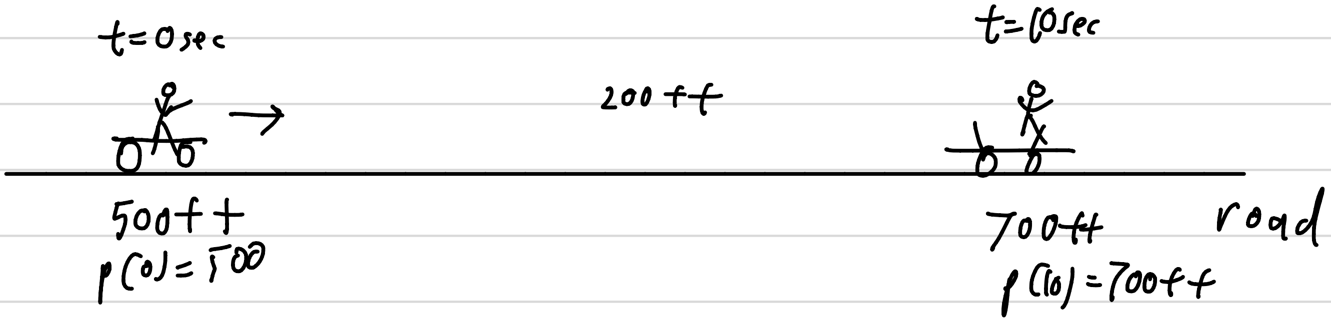
Displacement and Distance

Suppose an object is moving along a straight line. There is a difference between the "distance" the object travels, and the "displacement" of the object.

The "displacement" of an object over an interval of time can be positive or negative, and refers to the change in position of the object.

Suppose a bicycle travels along a straight road measured in feet. Let

$p(t) =$ The position of the bicycle (in feet) after t seconds



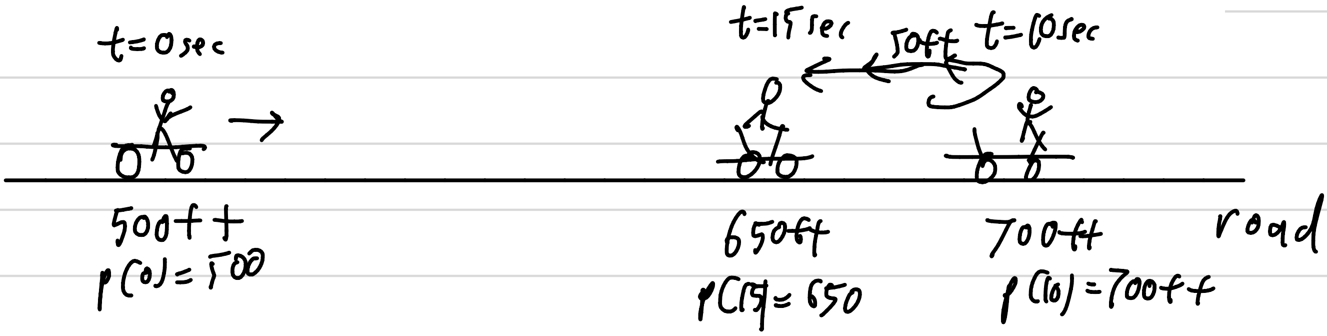
It starts at position 500 feet ($p(0) = 500$).

Then, from $t = 0$ seconds to $t = 10$ seconds, it travels 200 feet to the right (in the positive direction).

The displacement of the bicycle from $t = 0$ seconds to $t = 10$ seconds is 200 feet, because it traveled 200 feet to the right.

Suppose a bicycle travels along a straight road measured in feet. Let

$p(t) =$ The position of the bicycle (in feet) after t seconds



It starts at position 500 feet ($p(0) = 500$).

Then, from $t = 0$ seconds to $t = 10$ seconds, it travels 200 feet to the right (in the positive direction).

From t=10 sec to t=15 sec the bicycle travels 50 ft left.

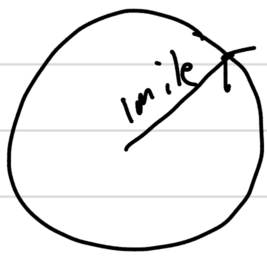
The displacement of the bicycle from t=10 sec to t=15 sec is −50ft = p(15)−p(10) = 650ft−700ft = −50ft

However the distance traveled from t=10 sec to t=15 sec is +50ft. Distance traveled is always positive.

Over the interval from 0 to 15 seconds the displacement of the bike is 200ft − 50ft = 150ft = p(15) − p(0) = 650ft − 500ft = 150ft

Over the same interval, the distance travelled is 200ft + 50ft = 250ft.

If I travel in a circle



my displacement is 0 miles,
I travel a distance 2π miles,

Suppose a bicycle is travelling along a straight road. The ~~velocity~~ of the bicycle is given by

position & velocity

$$p(t) = \text{The position of the bicycle (ft) at time } t \text{ sec}$$

$$p'(t) = v(t) = \text{The velocity of the bicycle (ft/sec) at time } t \text{ (sec)}$$

The velocity of the bicycle is measured every 10 seconds and recorded in a table:

t	0	10	20	30	40	50	60
$v(t)$	20	16	10	4	-2	-6	-10

$\Delta t = 10\,\text{sec}$

We want to approximate the displacement of the bicycle from 0 to 60 seconds.

We do this by dividing the time interval from 0 to 60 seconds into $n = 6$ sub-intervals of length $\Delta t = 10$ seconds each, and estimating the displacement of the bike over each 10 ~~6~~ second sub-interval.

	LH sum	RH sum
From 0 to 10 seconds, the bike travels approximately:	$20\frac{ft}{sec}\cdot 10sec = 200ft$	$16\frac{ft}{sec}\,10sec = 160ft$
From 10 to 20 seconds, the bike travels approximately:	$16\frac{ft}{sec}\,10sec = 160ft$	$10\frac{ft}{sec}\,10sec = 100ft$
From 20 to 30 seconds, the bike travels approximately:	100ft	40ft
From 30 to 40 seconds, the bike travels approximately:	40ft	-20ft
From 40 to 50 seconds, the bike travels approximately:	-20ft	-60ft
From 50 to 60 seconds, the bike travels approximately:	-60ft	-100ft

The total displacement of the bicycle from 0 to 60 seconds is approximately: 420ft (LH), 120ft (RH)

Conclusion. If

$$v(t) = \text{ The velocity of an object at time } t$$

Then

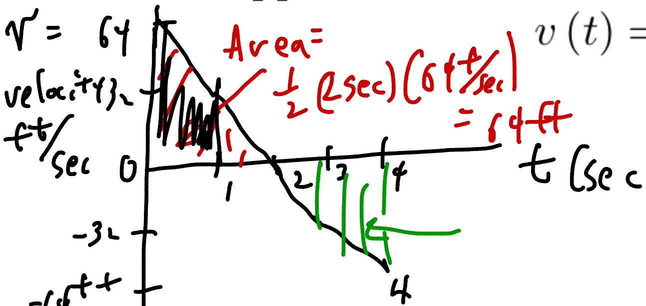
$$\int_a^b v(t)\,dt = \text{The displacement of the object from time } t = a \text{ to time } t = b$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} v(t_k^*)\Delta t = \lim_{n\to\infty} v(t_1^*)\Delta t + v(t_2^*)\Delta t + \dots$$

$v(t) =$ The velocity of the ball (in ft/sec upwards) at time t(sec)

and suppose

$$v(t) = 64 - 32t$$



t (sec)	v(t) (ft/sec)
0	64 (upwards at 64 ft/sec
1	32 ft/sec
2	0 (ball has stopped)
3	-32 ft/sec
4	-64 ft/sec

What is the displacement of the ball from $t = 0$ seconds to $t = 2$ seconds?

$$\int_0^2 v(t)\,dt = 64 \text{ ft}$$

What is the displacement of the ball from $t = 2$ seconds to $t = 4$ seconds?

$$\int_2^4 v(t)\,dt = -64 \text{ ft}$$

What is the displacement of the ball from $t = 0$ seconds to $t = 4$ seconds?

The Fundamental Theorem of Calculus

If $p'(t) = v(t)$

Then the displacement of an object from $t=a$ to $t=b$

is $$\int_a^b v(t)\,dt = p(b) - p(a)$$