Math 1b (8:30AM) 13 Jan 2020

If $v(t)$ = The velocity (in ft/sec) of some object at time t (sec)

Then

$$\int_a^b v(t)\,dt = \text{The } \overbrace{\text{displacement}}^{\text{change in position}} \text{(ft) of the object at time } t \text{ (sec)}$$

$$\int_a^b v(t)\,dt = \lim_{n\to\infty} \underbrace{v(t_1^*)\Delta t}_{\substack{\approx p(t_1)-p(t_0) \\ \text{approximately equal to displacement} \\ \text{over interval } [t_0, t_1]}} + \underbrace{v(t_2^*)\Delta t}_{\substack{\approx p(t_2)-p(t_1) \\ \text{approximately equal to displacement} \\ \text{over interval } [t_1, t_2]}} + v(t_3^*)\Delta t + \dots + v(t_n^*)\Delta t = \lim_{n\to\infty}\sum_{k=1}^{n} v(t_k^*)\Delta t$$

$$\Delta t = \frac{b-a}{n}$$

$$\text{displacement} = \int_a^b \text{velocity}$$

The distance travelled by the object from $t=a$ to $t=b$ is $\int_a^b |v(t)|\,dt$

$$\text{distance travelled} = \int_a^b \text{speed}$$

(assuming $a<b$)

$$\int_a^b f(t)\,dt = \lim_{n\to\infty}\sum_{k=1}^{n} f(t_k^*)\Delta t = \lim_{n\to\infty}\sum_{k=1}^{n} f(t_k^*)\frac{b-a}{n}$$

where $t_{k-1} \le t_k^* \le t_k$

that is $a+(k-1)\cdot\frac{b-a}{n} \le t_k^* \le a+k\frac{b-a}{n}$

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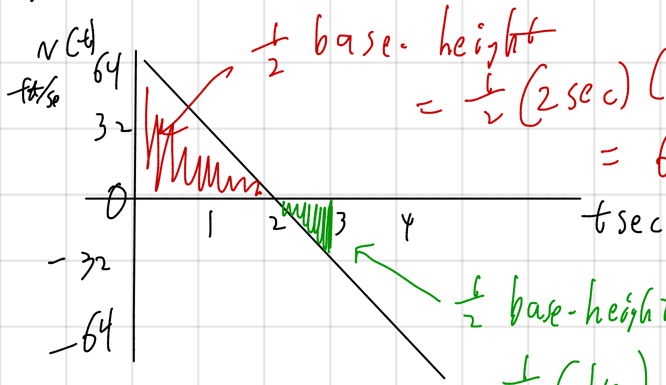
I throw a ball vertically up into the air

$v(t)$ = The velocity of ball (ft/sec upward) at time t (sec)

$v(t) = 64 - 32t$

t	$v(t)$
0	64 ft/sec
1	32 ft/sec
2	0 ft/sec
3	-32 ft/sec
4	-64 ft/sec



$\frac{1}{2}$ base $\cdot$ height

$= \frac{1}{2}(2\text{ sec})(64\text{ ft/s})$

$= 64\text{ ft}$

$\frac{1}{2}$ base $\cdot$ height

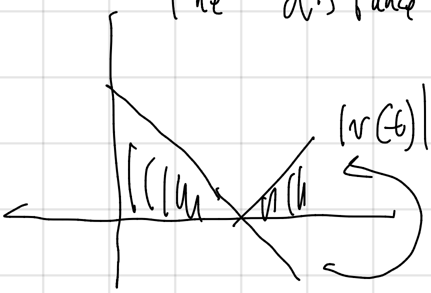
$\frac{1}{2}(1\text{ sec})(-32\text{ ft/sec})$

$= -16\text{ ft}$

Displacement of ball from $t=0$ sec to $t=3$ sec

is $\int_0^3 v(t)\,dt = 64 - 16 = 48$ ft

The distance the ball travels from $t=0$ sec to $t=3$ sec



is $\int_0^3 |v(t)|\,dt = 64 + 16 = 80$ ft

Suppose $F(t)$ = The amount of some quantity at time t

$F(t)$ could be total grams of a yeast population
$F(t)$ " " temperature °F
$F(t)$ " " the price of milk

Suppose $f(t)$ = rate of change of that quantity at time t

$$f(t) = F'(t)$$

Then
the net change in the quantity from $t=a$ to $t=b$

$$F(b)-F(a) = \int_a^b f(t)\,dt$$

$$\int_a^b f(t)\,dt = F(b) - F(a)$$

The integral of a rate of change is the resulting net change

$F(t)$ = The temperature (°F) at De Anza t hours after midnight on Jan 1, 2010.

$F(13) = 72$ means the temperature at De Anza at 1:00PM (Jan 1, 2010) was 72°F.

$f(t)$ = The rate the temperature is changing (°F/hr) at De Anza t hours after midnight, Jan 1, 2010.

$f(13) = -0.6$ means

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$f(13) = -0.6$ means At 1:00PM Jan 1, 2010, the temperature at De Anza was <u>decreasing</u> at a (instaneous) rate of 0.6°F/hr

$$F'(t) = f(t)$$

Suppose $\int_6^{12} f(t)\,dt = 3.4$

What does this mean?

From 6:00 AM to 12:00PM on Jan 1, 2010, the temperature increased a net amount of 3.4°F.

If at 6:00 AM the temperature was 60°F, what was the temperature at 12:00 PM?

63.4°F

$$\int_a^b f(t)\,dt = \lim_{n\to\infty}\Big[\overbrace{f(t_1^*)\Delta t}^{\approx F(t_1)-F(t_0)} + \overbrace{f(t_2^*)\Delta t}^{\approx F(t_2)-F(t_1)} + \dots + f(t_n^*)\Delta t\Big]$$

approximates the amount the temperature changed from $t=t_0$ to $t=t_1$

approximates the amount the temperature changed from $t=t_0$ to $t=t_1$

The fundamental Theorem of calculus part II

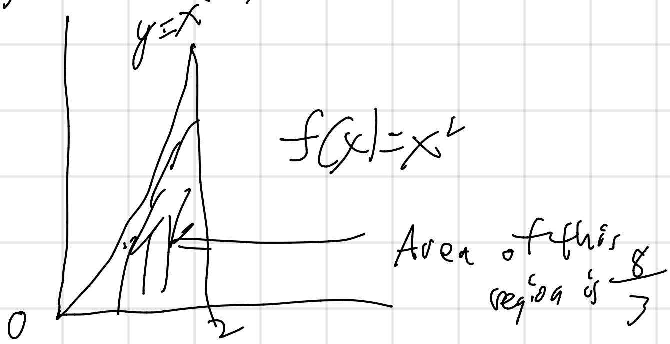
(Most books call this part I, but our book calls it part II, so I will too).

$$\boxed{\begin{array}{l} \text{If } F'(t) = f(t) \text{ on an interval } [a,b] \\ \text{then} \\ \displaystyle\int_a^b f(t)\,dt = F(b) - F(a) \\ \text{(provided } f \text{ is continuous)} \end{array}}$$

What is the area of the region given by

$0 \le y \le x^2$

$0 \le x \le 2$



$$\int_0^2 x^2\,dx = \int_0^2 f(x)\,dx = F(2) - F(0) = \frac{1}{3}2^3 - \frac{1}{3}0^3$$

$$= 8/3$$

Find $F(x)$, $F'(x) = f(x)$, $F'(x) = x^2$

$$F(x) = \frac{1}{3}x^3$$