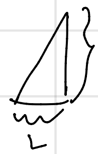
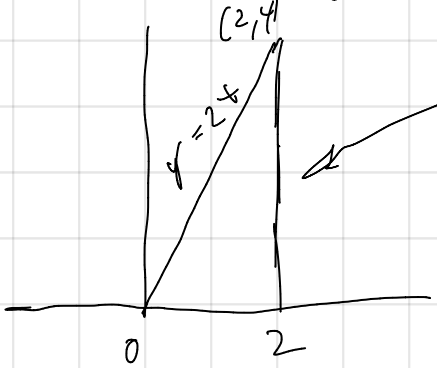
Math 1b (8:30AM) 14 Jan 2020

The Fundamental Theorem of Calculus, Part 2 If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where F is any antiderivative of f, that is, a function such that $F' = f$.

$\int_0^2 2t\,dt =$ Area of triangle  $= \frac{1}{2}\, 2 \cdot 4 = 4$



Let $f(t) = 2t$

$$\int_0^2 2t\,dt = \int_0^2 f(t)\,dt$$

Find $F(t)$, $F'(t) = f(t) = 2t$

$$F(t) = t^2$$

$$\int_0^2 2t\,dt = \int_0^2 f(t)\,dt = F(2) - F(0)$$

$$= 2^2 - 0^2 = 4$$

We want to introduce new notation that makes it easier to apply the FTC (Fundamental theorem of calculus).

$$F(x)\Big|_a^b = F(b) - F(a)$$

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$$F(x)\Big|_a^b = F(b)-F(a)$$

"F(x) evaluated from a to b"

$$F(x)\Big|_{x=a}^{x=b} = F(b)-F(a)$$

$$\Big[F(x)\Big]_a^b = F(b)-F(a)$$

$$\Big[F(x)\Big]_{x=a}^{x=b} = F(b)-F(a)$$

$$x^2\Big|_2^3 = 3^2-2^2=5 \qquad \Big[x^2-x\Big]_1^2 = (2^2-2)-(1^2-1) = 2-0 = 2$$

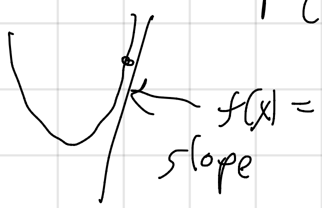
In this new notation, we can write the FTC as

$$\int_a^b f(x)\,dx = \overbrace{F(x)\Big|_a^b}^{F(b)-F(a)}$$

$F(x) = x^2$

$$\int_0^2 2t\,dt = t^2\Big|_0^2 = 2^2-0^2 = 4$$

$f(x) = 2x$
$= F'(x)$



If $f(t) = 2t$, then $f(t)$ has many antiderivative, $F_1(t) = t^2$, $F_2(t) = t^2+17$ $F_3(t) = t^2-42\ldots$

The general antiderivative is $F(t) = t^2+C$

But in evaluating a definite integral we don't need the $+C$.

$$\int_0^2 2t\,dt = \left[t^2+17\right]_0^2$$

$$= (2^2+17) - (0^2+17) = 2^2 - 0^2 = 4$$

$$\int_0^2 2t\,dt = t^2\Big|_0^2$$

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where F is any antiderivative of f, that is, a function such that $F' = f$.

Most books call this theorem Part I of the FTC. Our book reverses the numbering and calls it Part II.

We will not be studying Part I of FTC until much later in the course.

When you are reading the text, section 5.3, you can skip part I of the FTC for now and go straight to Part II.

5.4

We use anti-derivatives to compute definite integrals.

Therefore it would be good to introduce a new more efficient notation for calculating anti-derivatives.

This new notation is called the "indefinite integral."

We indicate an indefinite integral by writing an integral sign

$$\int$$

without any limits of integration.

an indefinite integral $\longrightarrow$

$$\int f(t)\,dt = F(t) + c$$

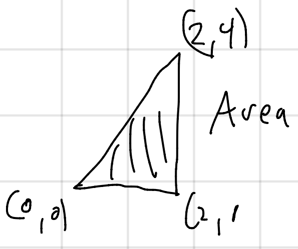
This means $F(t)$ is an anti derivative of $f(t)$.

It means $\frac{d}{dt}F(t) = f(t)$

$$\int 2t\,dt = t^2 + c$$

Definite Integral

$$\int_0^2 2t\,dt = \underset{\text{A number}}{4}$$



$$\int_0^2 2t\,dt = t^2\Big|_0^2 = 2^2 - 0^2 = 4$$

Indefinite integral

$$\int 2t\,dt = \underbrace{t^2 + c}_{\text{a function}}$$

(a family of functions)

A lot of this course is dedicated to calculating anti-derivatives

sections 5.4, 5.5, 7.1, 7.2, 7.3, 7.4, 7.5

$$\int 17\,dx = \underbrace{17x} + C$$

$$\frac{d}{dx} 17x = 17$$

First Rule: $\int \overset{\text{constant}}{K}\,dx = Kx + C$

$$\int K\,dt = Kt + C$$

Power Rule

$$\int x^{\overset{n \text{ is a constant}}{n}}\,dx$$

$$\frac{d}{dx} x^{\overset{n \text{ is a constant}}{n}} = n x^{n-1}$$

$$\frac{d}{dx} x^5 = 5x^4$$

$\frac{d}{dx} e^{x}$ ← not a constant. We do not use power rule.

$$\boxed{\int x^n\,dx = \frac{1}{n+1} x^{n+1} + C}$$

$$\Downarrow$$

$$\frac{d}{dx}\left(\frac{1}{n+1} x^{n+1}\right) = \frac{1}{n+1}(n+1)\, x^n \Leftarrow \text{should be } x^n$$

$$= x^n \checkmark$$

$$\boxed{\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C}$$

$$\Downarrow$$

$$\frac{d}{dx}\left(\frac{1}{n+1}x^{n+1}\right) = \frac{1}{n+1}(n+1)\,x^n \Leftarrow \text{should be } x^n$$

$$= x^n \ \checkmark$$

$$\int x^{n+1}\,dx = \frac{1}{n+1}x^{n+1} + C$$

because $\frac{d}{dx}\left(\frac{1}{n+1}x^{n+1}\right) = \frac{1}{n+1}(n+1)\,x^n = x^n \ \checkmark$

+1 recip

$$\int x^5\,dx = \frac{1}{6}x^6 + C$$

+1 recip

$$\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx = \frac{2}{3}x^{3/2} + C$$

+1 recip

$$\int \frac{1}{x^2}\,dx = \int \frac{dx}{x^2} = \int x^{-2}\,dx = -x^{-1} + C = \frac{-1}{x} + C$$

$$\int \frac{1}{x}\,dx = \int x^{-1}\,dx = \cancel{\frac{1}{0}x^0}$$ Does not work

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$$

provided n is a constant and $n \neq -1$.

$$\int \frac{1}{x}\,dx = \ln|x| + C \qquad \frac{d}{dx}\ln|x| = \frac{1}{x}$$

$$\int x^n\,dx = \begin{cases} \frac{1}{n+1} x^{n+1} + C, & n \neq -1 \\ \ln|x| + C, & n = -1 \end{cases}$$

$$\int cf(x)\,dx = c\int f(x)\,dx \qquad \int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int k\,dx = kx + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1) \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C \qquad \int b^x\,dx = \frac{b^x}{\ln b} + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C \qquad \int \csc x \cot x\,dx = -\csc x + C$$

$$\int \frac{1}{x^2+1}\,dx = \tan^{-1}x + C \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$$

$$\int \sinh x\,dx = \cosh x + C \qquad \int \cosh x\,dx = \sinh x + C$$