Math 1b (8:30AM) 15 Jan 2020

indefinite integral

$$\int f(x)\,dx = F(x)+c$$

means $\frac{d}{dx}F(x) = f(x)$

or $dF(x) = f(x)\,dx$

definite integral

$$\int_a^b f(x)\,dx = L$$

means

$$L = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

$\int 2x\,dx = x^2+c$ because $\frac{d}{dx}x^2 = 2x$ (or $dx^2 = 2x\,dx$)

constant funct

$\int 7\,dx = 7x+c$ because $\frac{d}{dx}7x = 7$

$\int k\,dx = kx+c$ because $\frac{d}{dx}kx = k$

Definite integral

$\int_a^b k\,dx = k\cdot(b-a)$

$\int_6^9 7\,dx = 7(9-6) = 21$

$$\int x^n\,dx = \begin{cases} \frac{1}{n+1}x^{n+1}+c, & n\neq -1 \\ \ln|x|+c, & n=-1 \end{cases}$$

$\int x^3\,dx = \frac{1}{4}x^4 + c$ (+1)

$\int \frac{1}{x}\,dx = \ln|x|+c$

$\int 20x^3\,dx = 20\cdot\frac{1}{4}x^4 + c = 5x^4 + c$ ✓

$\frac{d}{dx}5x^4 = 5\cdot 4x^3 = 20x^3$

Constant multiple rule for indefinite integral

constant

$$\int k f(x)\,dx = k\int f(x)\,dx$$

Proof

Suppose $\int f(x)\,dx = F(x)+c$

then $\frac{d}{dx}F(x) = f(x)$, by the constant multiple for derivative,

$$\frac{d}{dx}kF(x) = k\frac{d}{dx}F(x) = kf(x)$$

Constant multiple rule for indefinite integral

$$\int \underset{\text{constant}}{k}\, f(x)\,dx = k \int f(x)\,dx$$

Proof

Suppose $\int f(x)dx = F(x) + c$

then $\frac{d}{dx} F(x) = f(x)$, by the constant multiple for derivative,

$$\frac{d}{dx} kF(x) = k \frac{d}{dx} F x = kf(x)$$

and so

$$\int k f(x) dx = k F(x) + C$$

$$= k \int f(x) dx$$

$$\boxed{\int k f(x) dx = k \int f(x) dx}$$

$$\boxed{\int_a^b k f(x) dx = k \int_a^b f(x) dx}$$

$$\int (f(x) + g(x)) dx = \int f(x) dx + \int g(x) dx$$

(provided $\int f(x)dx$ and $\int g(x) dx$ both exist)

Definite Integrals

$$\int_a^b (f(x) + g(x)) dx$$

$$= \int_a^b f(x) dx + \int_a^b g(x) dx$$

An anti-derivative of a sum is the sum of the anti-derivatives

$$\int (4x - 3)\, dx$$

$$= \int 4x\, dx - \int 3 dx$$

$$= \boxed{2x^2 - 3x + c}$$

check: $\frac{d}{dx}(2x^2 - 3x) = 4x - 3$ ✓

There is no product/quotient Rule for anti-derivatives.

WRONG

$$\int x\,(x^2+x^3)\,dx = \cancel{\left(\tfrac{1}{2}x^2\right)\left(\tfrac{1}{3}x^3+\tfrac{1}{4}x^4\right)+C}$$

$$\int f(x)g(x)\,dx \neq \left(\int f(x)\right)\left(\int g(x)\,dx\right)$$

$$\int x\,(x^2+x^3)\,dx = \int (x^3+x^4)\,dx$$

$$= \tfrac{1}{4}x^4 + \tfrac{1}{5}x^5 + C$$

$$(fg)' = \cancel{f'g'}$$

$$(fg)' = f'g + fg'$$

$$\frac{d}{dx} e^x = e^x \qquad \int e^x dx = e^x + C$$

functions | co-functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x \qquad \int \cos x\,dx = \sin x \qquad \int \sin x\,dx = -\cos x + C$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\cot x = -\csc^2 x \qquad \int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \frac{d}{dx}\csc x = -\csc x \cot x \qquad \int \sec x \tan x\,dx = \sec x + C \qquad \int \csc x \cot x\,dx = -\csc x + C$$

$$\int cf(x)\,dx = c\int f(x)\,dx \qquad \int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int k\,dx = kx + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1) \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C \qquad \int b^x\,dx = \frac{b^x}{\ln b} + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C \qquad \int \csc x \cot x\,dx = -\csc x + C$$

$$\int \frac{1}{x^2+1}\,dx = \tan^{-1} x + C \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$$

$$\int \sinh x\,dx = \cosh x + C \qquad \int \cosh x\,dx = \sinh x + C$$

My favourite derivative / anti-derivative

$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2} \qquad \int \frac{1}{1+x^2}\,dx = \arctan x + C$$

$$\int \frac{dx}{1+x^2} = \arctan x + C$$

$$\int cf(x)\,dx = c\int f(x)\,dx \qquad \int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int k\,dx = kx + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1) \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C \qquad \int b^x\,dx = \frac{b^x}{\ln b} + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C \qquad \int \csc x \cot x\,dx = -\csc x + C$$

$$\int \frac{1}{x^2+1}\,dx = \tan^{-1}x + C \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$$

$$\int \sinh x\,dx = \cosh x + C \qquad \int \cosh x\,dx = \sinh x + C$$

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$$

$$\frac{d}{dx}\arccos x = \frac{-1}{\sqrt{1-x^2}} \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = -\arccos x + C$$

We don't need this because we already have a rule for $\int \frac{1}{\sqrt{1-x^2}}\,dx$

$$\arcsin x + \arccos x = \frac{\pi}{2}$$

$$\underline{\arcsin x} = \underline{-\arccos x} + \frac{\pi}{2}$$

$$\frac{d}{dx}\sinh x = \cosh x \qquad \frac{d}{dx}\cosh x = \sinh x$$

hyperbolic trig functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\int \sinh\,dx = \cosh x + C \qquad \int \cosh x\,dx = \sinh x + C$$

$$\frac{d}{dx} e^x = e^x$$

$$\int e^x \, dx = e^x + C$$

b is a constant

$$\frac{d}{dx} b^x = (\ln b)\, b^x$$

$$\int b^x \, dx = \frac{1}{(\ln b)} b^x + C$$

check:

$$\frac{d}{dx} \frac{1}{\ln b} b^x$$

$$\cancel{\frac{1}{\ln b}} \cancel{(\ln b)}\, b^x$$

$$= b^x$$

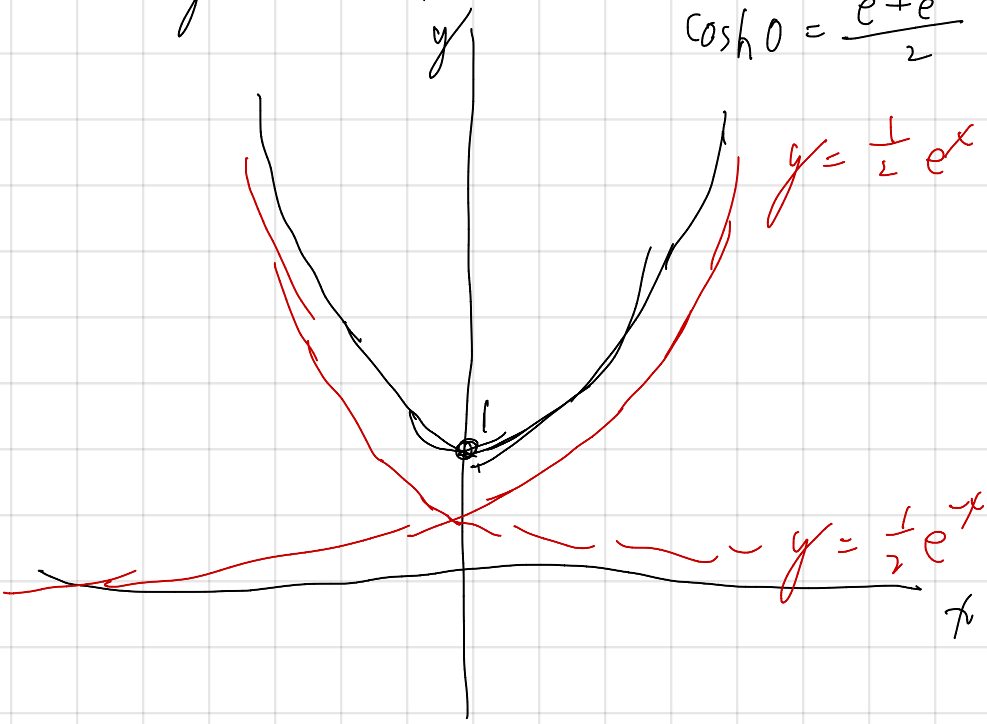
$$\frac{d}{dx} 2^x = (\ln 2)\, 2^x$$

$$\frac{d}{dx} 17^x = (\ln 17)\, 17^x$$

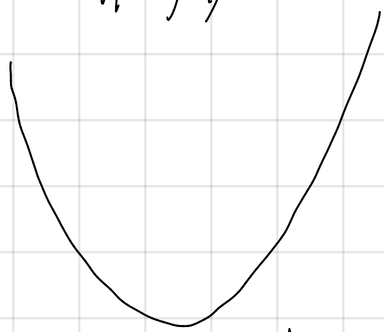
$$\frac{d}{dx} e^x = \underbrace{(\ln e)}_{=1} e^x = e^x$$

$$y = \cosh x = \frac{e^{x}+e^{-x}}{2} = \frac{1}{2}e^{x} + \frac{1}{2}e^{-x}$$

$$\cosh 0 = \frac{e^{0}+e^{-0}}{2}$$



Hanging Chain



catenoid

$$y = \sinh x = \frac{e^{x} - e^{-x}}{2}$$