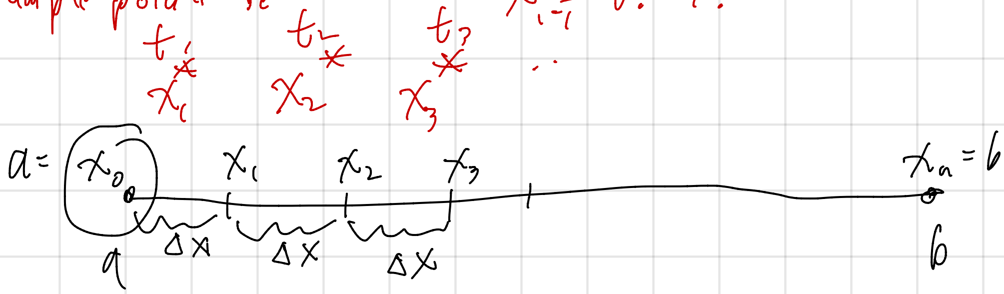
Math 16 (8:30AM) 16 Jan 2020

We could have said
Let sample points be

$x_{i-1} \le t_i \le x_i$

$$\int_a^b f(x)\,dx$$



n subdivision

$$x_i = a + i\Delta x$$

$$x_{i-1} \le x_i^* \le x_i$$

5.5/ Integration by substitution is used to integrate compositions of functions (just like the chain rule was used to integrate ~~com~~ derivatives of compositions)

$$\int \cos u \, du = \sin u + C$$

$u = x^2$

$$\int \cos(x^2)\, dx = \cancel{\sin(x^2) + c}$$

You might guess the answer is

but this is wrong

$$\frac{d}{dx}\underline{\sin(x^2)} = \underline{2x \quad \cos x^2}$$

$$\frac{d}{dx} f(g(x)) = g'(x)\, f'(g(x))$$

$$\int 2x \cos(x^2)\, dx = \sin x^2 + C$$

$$\int f(u)\, du = F(u) + C \qquad F'(u) = f(u)$$

$$\frac{d}{dx} F(g(x)) = g'(x) F'(g(x)) = \underline{g'(x) f(g(x))}$$

Therefore if $\int f(u)\, du = F(u) + C$

Then $\int g'(x)\, f(g(x))\, dx = F(g(x)) + C$

$$\int \overset{f}{\widehat{\cos}} u \, du = \overset{F}{\widehat{\sin}} u + C$$

$$\int \overset{g'(x)}{2x} \cos \overset{g(x)}{\widetilde{x^2}}\, dx = \sin x^2 + C$$

Therefore if $\int f(u)\,du = F(u) + C$

Then $\int g'(x) f(g(x))\,dx = F(g(x)) + C$

$$\int \underbrace{\cos u}_{f}\,du = \underbrace{\sin u}_{F} + C$$

$$\int \underbrace{2x}_{g'(x)} \cos \underbrace{x^2}_{g(x)}\,dx = \sin x^2 + C$$

If $\int f(u)\,du = F(u) + C$

Let $u = g(x)$ Then $du = g'(x)\,dx$

Then $\int f(g(x))\,\underbrace{g'(x)\,dx}_{du} = F(\underbrace{g(x)}_{u}) + C$

$$\int \underbrace{2x \; dx}_{2x\,dx} \cos x^2 = \int \cos u \; \underbrace{du}_{du} = \sin u + C$$

Let $u = x^2$, then $du = 2x\,dx$

$= \sin x^2 + C$

$$\int g'(x)\, f\,[g(x)]\,dx$$

Look for something $u = g(x)$
inside of a function whose derivative
$g'(x)dx = du$

$$\int \frac{e^{1/x}}{x^2}\,dx \qquad \frac{1}{x^2}dx$$

$u = \frac{1}{x}$ $\quad du = \frac{-1}{x^2}dx$

$-du = \frac{1}{x^2}dx$

$$= \int e^u(-du) = -e^u + C = -e^{\frac{1}{x}} + C$$

$$\int x \cos x^2\, dx$$

$u = x^2$ $\quad du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$= \int (\cos u)\,\tfrac{1}{2}\,du = \tfrac{1}{2}\sin(u) + C = \boxed{\tfrac{1}{2}\sin x^2 + C}$$

check $\frac{d}{dx}\frac{1}{2}\sin x^2 + \frac{1}{2}(\cos x^2)\,2x = x\cos x^2$ ✓

Another way to do $\int x \cos x^2 dx$ $\quad u = x^2 \quad du = 2x\,dx$

$$= \frac{1}{2}\int 2x \cos x^2\, dx \qquad 2x\,dx = du$$

$$= \frac{1}{2}\int \cos u\ du = \frac{1}{2}\sin u + C = \boxed{\frac{1}{2}\sin x^2 + C}$$

Problem: $\int \cos x^2 \, dx = ?$

I don't know how to do this.

A very common error

$$\int \frac{1}{u} \, du = \ln|u| + C$$

$$\int \frac{1}{1+x^3} \, dx = \cancel{\ln|1+x^3| + C}$$

WRONG

$$\int \frac{3x^2}{1+x^3} \, dx =$$

$u = 1+x^3 \qquad du = 3x^2 \, dx$

$$= \int \frac{1}{u} \, du = \ln|u| + C = \ln|1+x^3| + C$$

$du = f'(x)dx$

$$\int \frac{f'(x)}{f(x)} \, dx = \ln|f(x)| + C$$

$u = f(x)$

$$= \int \frac{1}{u} \, du = \ln|u| + C = \ln|f(x)| + C$$

$$\int \frac{3x^2}{1+x^3} \, dx = \ln|1+x^3| + C$$

$$\int \frac{x^2}{1+x^3} \, dx = \frac{1}{3} \int \frac{3x^2}{1+x^3} \, dx = \frac{1}{3} \ln|1+x^3| + C$$

$$\int 2x\sqrt{9+x^2}\,dx$$

$$\int 2x\sqrt{9+x^2}\,dx \qquad u = 9+x^2 \qquad du = 2x\,dx$$

$$= \int \sqrt{u}\,du = \frac{2}{3}u^{3/2}+C = \frac{2}{3}(9+x^2)^{3/2}+C$$

$u^{1/2}$

$$\int 2x\sqrt{9+x^2}\,dx = \frac{2}{3}(9+x^2)^{3/2}$$

Substitution for definite integrals

$$\int_0^4 2x\sqrt{9+x^2}\,dx$$

Method #1

$u = 9+x^2$

$du = 2x$

$$\int_0^4 2x\sqrt{9+x^2}\,dx$$

A very common WRONG

$$\cancel{= \int_0^4 \sqrt{u}\,du}$$

Mistake

$u = x^2+9$

$$\int_{x=0}^{x=4} 2x\sqrt{9+x^2}\,dx = \int_{u=0^2+9=9}^{u=4^2+9=25} \sqrt{u}\,du = \frac{2}{3}u^{3/2}\Big|_9^{25} = \frac{250}{3} - \frac{54}{3} = \frac{196}{3}$$

Method #2

$$\frac{2}{3}(9+x^2)^{3/2}\Big|_0^4$$

$$\frac{2}{3}(9+4^2)^{3/2} - \frac{2}{3}(9+0^2)^{3/2}$$

$$\frac{250}{3} - \frac{54}{3} = \frac{196}{3}$$