Math 1b (8:30 AM) 17 Jan 2020

What are dx, dy, $\frac{dy}{dx}$ what are they all about?

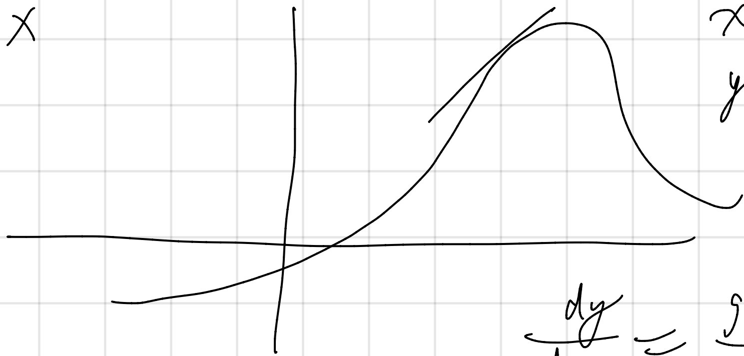
I think of x, y, z as being functions

dx, dy, dz represent the derivatives of functions

$d\,x^3$ ← Is a composition of functions. Composition of cubing with x

$dx^3 = 3x^2\,dx$

If $y = x^3$ $\frac{dy}{dx} = \frac{dx^3}{dx} = 3x^2$



$x = f(t)$
$y = g(t)$

$\frac{dy}{dx} = \frac{g'(t)}{f'(t)}$

① $\int \frac{t}{1+t^2}\,dt$

$= \frac{1}{2}\ln(1+t^2)+C$

② $\int \frac{1}{1+t^2}\,dt$

$= \arctan t + C$

The derivative of $1+t^2$

$\int \frac{t}{1+t^2}\,dt$

inner functio

In doing a u substitution, look for an inner function to be u, that has its derivate outside to be du

$u = 1+t^2 \qquad du = 2t\,dt$

$\frac{1}{2}\int \frac{2t}{1+t^2}\,dt = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln(1+t^2) + C$

$t^2 = u \qquad 2t\,dt = du$

$\int t e^{t^2}\,dt = \int \frac{1}{2} e^u\,du = \frac{1}{2}e^u + C$

$= \frac{1}{2}e^{t^2} + C$

Reverse substitution

$$\int x \sqrt{2x+5}\, dx$$

?

$$u = 2x+5$$

$$du = 2\,dx$$

reverse substitution

solve for x in terms of u

$$2x = u-5$$

$$x = \frac{1}{2}u - \frac{5}{2}$$

$$dx = \frac{1}{2}du$$

$$\int \left(\frac{1}{2}u - \frac{5}{2}\right)\sqrt{u}\,\frac{1}{2}\,du$$

$$= \frac{1}{4}\int u^{3/2}\,du - \frac{5}{4}\int u^{1/2}\,du$$

$$= \frac{1}{4}\cdot\frac{2}{5}u^{5/2} - \frac{5}{4}\cdot\frac{2}{3}u^{3/2} + C$$

$$= \frac{1}{10}(2x+5)^{5/2} - \frac{5}{6}(2x+5)^{3/2} + C \quad \checkmark$$

$$= \frac{\sqrt{2x+5}}{30}\left(3(2x+5)^2 - 25(2x+5)\right) + C$$

$$u = x \qquad du = dx$$

$$\int x\sqrt{2x+5}\,dx = \int u\sqrt{2u+5}\,du$$

Substitution in definite integrals

$$\int_1^5 \frac{x}{1+x^2}\,dx$$

$$u = 1+x^2$$
$$du = 2x\,dx$$
$$x\,dx = \frac{1}{2}\,du$$

when substituting in a definite integral, do not forget to change the limits of integration

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

x	$u=1+x^2$
1	2
5	26

$$\int_{x=1}^{x=5} \frac{x}{1+x^2}\,dx = \frac{1}{2}\int_{u=2}^{u=26} \frac{1}{u}\,du$$

$$= \frac{1}{2}\ln|u|\Big|_2^{26} = \frac{1}{2}\ln 26 - \frac{1}{2}\ln 2$$

$$= \frac{1}{2}\ln 13$$

Suppose

> $$\boxed{\int_0^{25} f(x)\,dx} = 42$$

Then what is

$$\int_0^5 x\,f(x^2)\,dx \; ?$$

x	$u=x^2$
0	0
5	25

> $$\int_0^5 x\,f(x^2)\,dx = \cancel{\frac{1}{2}\int_0^5 f(u)\,du}$$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$= \frac{1}{2}\boxed{\int_0^{25} f(u)\,du}$$

$$= \frac{1}{2}\int_0^{25} f(x)\,dx$$

$$= \frac{1}{2}(42) = 21$$

Suppose

$$\int f(x)\,dx = F(x) + c$$

Then

$$u = 3x+5, \quad du = 3\,dx, \quad dx = \tfrac{1}{3}\,du$$

$$\int f(3x+5)\,dx = \frac{1}{3}\int f(u)\,du = \frac{1}{3}F(u) + c$$

$$= \frac{1}{3}F(3x+5) + c$$

$$\int \cos x\,dx = \sin x + c \qquad \int f(3x+5)\,dx = \frac{1}{3}F(3x+5) + c$$

$$\int \cos(3x+5)\,dx = \frac{1}{3}\sin(3x+5) + c$$

If m, k are constants, so that $mx+k$ is a linear function of x,

then if $\int f(x)\,dx = F(x)+c$ the $\int f(mx+k)\,dx = \frac{1}{m}F(mx+k)+c$

$$\int x^{99}\,dx = \frac{1}{100}x^{100} + c$$

So $$\int (5x-2)^{99}\,dx = \frac{1}{5}\cdot\frac{1}{100}(5x-2)^{100} = \frac{1}{500}(5x-2)^{100}$$

$$\int e^{2x}\,dx = \frac{1}{2}e^{2x} \qquad \int e^{-x}\,dx = -e^{-x} + c$$

$$\int f(mx+k)\,dx = \frac{1}{m}F(mx+k)$$

I could have done this using a substitution $u = 2x$

$du = 2\,dx \quad dx = \frac{1}{2}\,du$ etc

$$\int e^{kx}\,dx = \frac{1}{k} e^{kx} + C$$

$$\int e^{x^2}\,dx$$

$u = x^2$ does not work because there is no $x\,dx$ to be my du.

This integral is impossible to solve using the techniques we learn in this class. This has been proven.