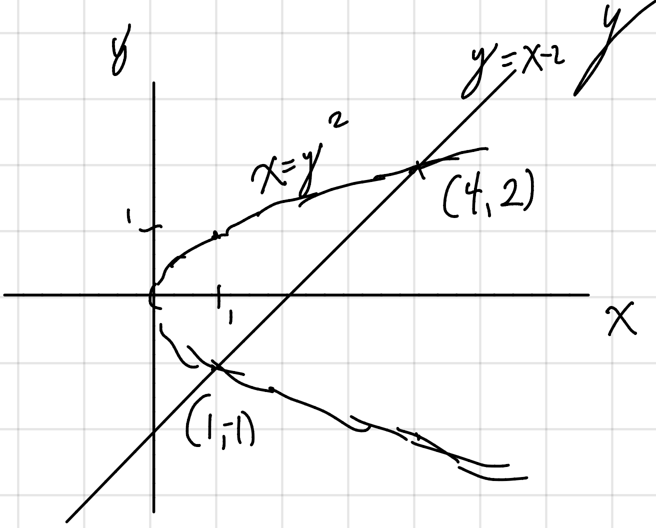
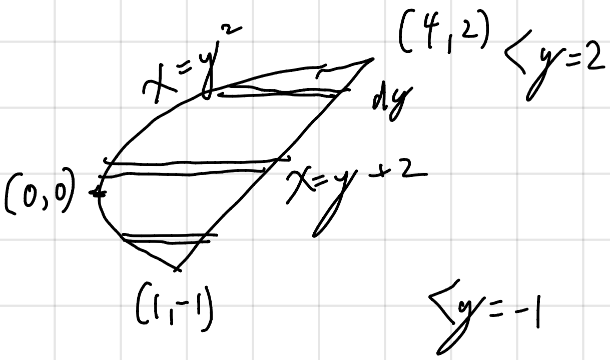
Math 1b (8:30 AM) 24 Jan 2020

Find the volume of the region between the curves

$x = y^2$
$y = x - 2$



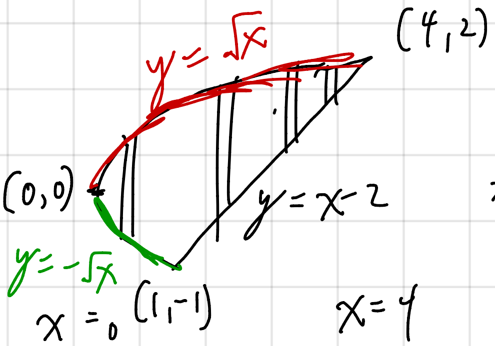


length

$$\int_{y=-1}^{y=2} \left(y+2-y^2\right) dy$$

$$= \left[\tfrac{1}{2}y^2 + 2y - \tfrac{1}{3}y^3\right]_{-1}^{2}$$

$$= \tfrac{3}{2} + 6 - \tfrac{1}{3}\cdot 9 = 4\tfrac{1}{2}$$



$$\int_{x=0}^{x=1} \underbrace{\left(\sqrt{x} - (-\sqrt{x})\right)}_{2\sqrt{x}} + \int_{x=1}^{x=4} \left(\sqrt{x} - (x-2)\right) dx$$

$$\tfrac{4}{3}x^{\frac{3}{2}}\Big|_0^1 + \left[\tfrac{2}{3}x^{\frac{3}{2}} - \tfrac{1}{2}x^2 + 2x\right]_1^4$$

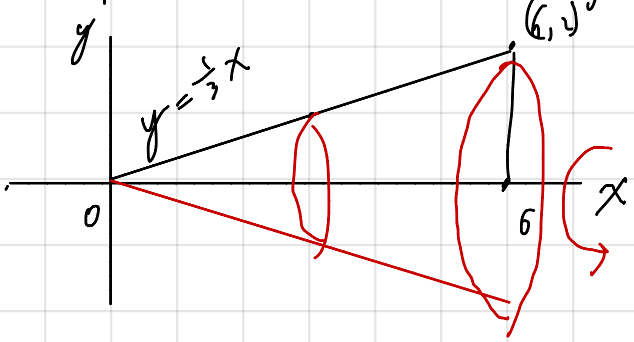
$$= \frac{4}{3} + \frac{2}{3}(7) - \frac{1}{2}(15) + 2\cdot 3$$

$$= \frac{18}{3} - \frac{15}{2} + 6 = 12 - \frac{15}{2}$$

$$= \frac{9}{2} = 4\tfrac{1}{2}$$

Section 6.2

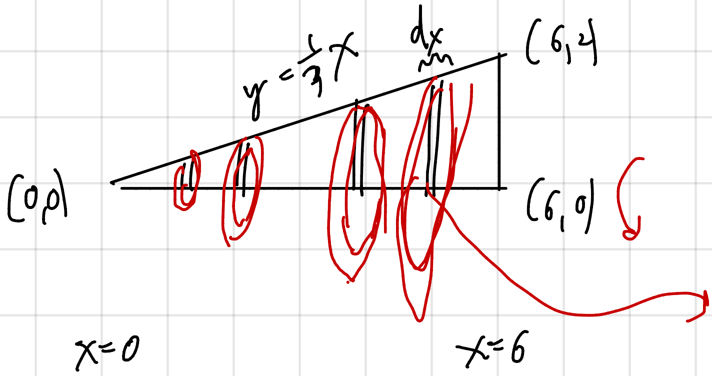
Suppose I take the region $0 \le x \le 6$ $0 \le y \le \frac{1}{3}x$



and I rotate it about the x-axis
It will sweep out a solid three dimensional shape, called a "solid of revolution.

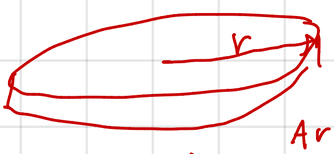
In this case we get a cone whose volume is $\frac{1}{3}$(Area of base)(height)
$= \frac{1}{3}(\pi\, 2^2)\, 6 = 8\pi$

We want to use calculus to find the volume of this solid.



Each rectangle rotated about the x-axis sweeps out a "disk."

A formula for the volume of each disk

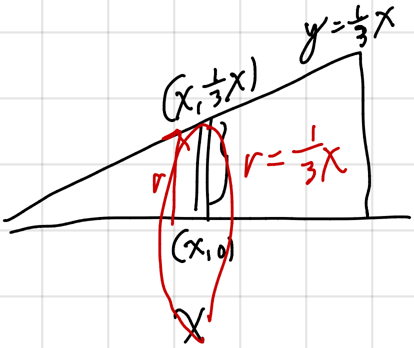


thickness $= dx$

Area
Volume of disk $= (\pi r^2)\, dx$

We use an integral to add the volumes of all the disks, to get the volume of the solid of revolution.

$$\text{Volume of solid} = \int_{x=0}^{x=6} \pi r^2\, dx$$



$$= \int_0^6 \pi \left(\frac{1}{3}x\right)^2 dx = \frac{\pi}{9}\int_0^6 x^2\, dx$$

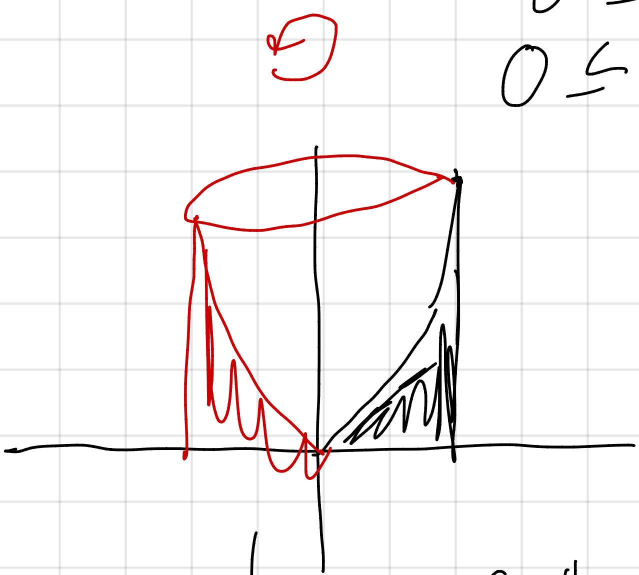
$$= \frac{\pi}{27} x^3 \Big|_0^6 = \underline{\underline{8\pi}}$$

The disk method

The washer method

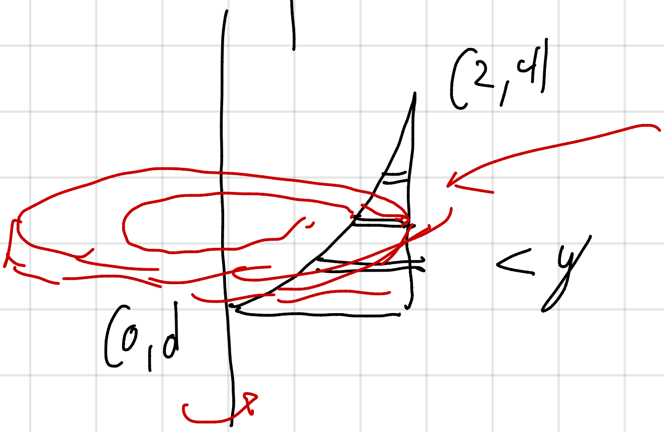
Take this region $0 \le x \ \ 2$

$0 \le y \le x^2$

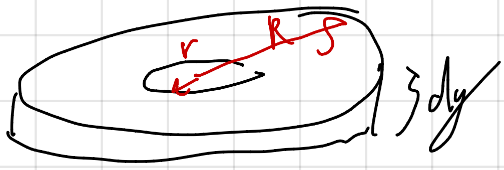


and rotate it about the y-axis

Find the volume of the resulting solid of revolution using an integral wrt, y



Rotating a horizontal rectangle about the y-axis sweeps out a disk with a hole in it, a "washer"



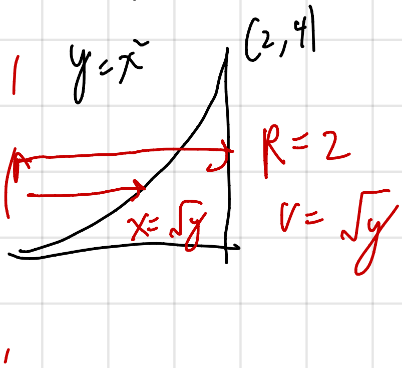
Formula for volume of washer

R = radius of outside of washer

r = radius of hole

Volume of washer = $\pi R^2 dy - \pi r^2 dy = \pi (R^2 - r^2) dy$

$$\int_{y=0}^{y=4} (\text{volume of the washer at vertical coordinate } y)$$



$$= \int_0^4 \pi (R^2 - r^2)\, dy$$

$$\pi \int_0^4 (2^2 - \sqrt{y}^2)\, dy$$