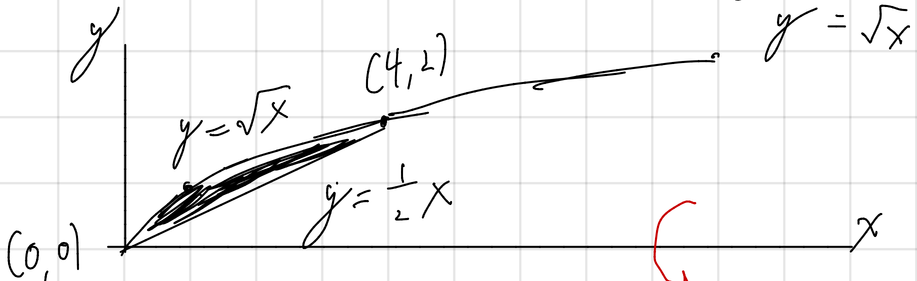
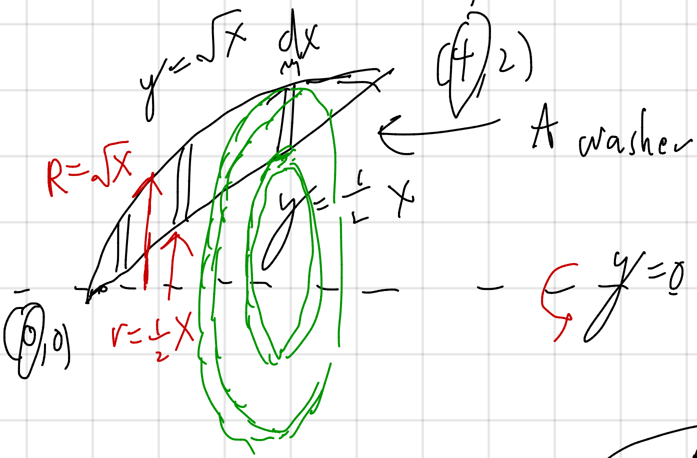
Math 1b (8:30AM) 27 Jan 2020

Take region between the curves

$$y = \frac{1}{2}x$$
$$y = \sqrt{x}$$

Rotate it about the x-axis and find the volume of the solid of revolution writing an integral(s) wrt x (i.e. "with respect to")





A washer (a disk with a whole in it)

volume of a washer

$$= \pi(R^2 - r^2)\,dx$$

Volume of solid of revolution $= \pi \int_{x=0}^{x=4} (R^2 - r^2)\,dx$

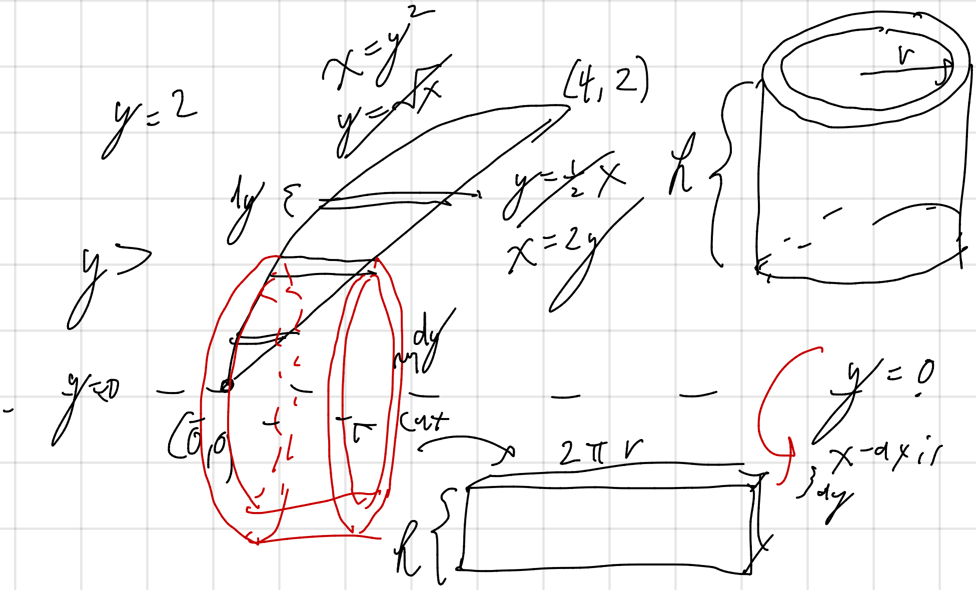
$$= \boxed{\pi \int_0^4 \left(x - \frac{1}{4}x^2\right) dx} = \pi\left[\frac{1}{2}x^2 - \frac{1}{12}x^3\right]_0^4$$

$$= \pi\left(8 - \frac{64}{12}\right) = \pi\left(\frac{24}{3} - \frac{16}{3}\right) = \frac{8}{3}\pi$$

Usually on exams or quizzes, I will ask you to stop here "Do not evaluate/simplify"

How can we do the same problem (Getting the same answer) using an integral wrt y instead of x?

(This will bring us into section 6.3)



Formula for the volume of shell.

$$V = (\text{Area of surface}) \cdot \text{Thickness}$$

$$= \underbrace{2\pi r}_{\text{circumference}}\ \underbrace{h}_{\text{height}}\ dy$$

When I rotate a rectangle about an axis parallel to it, I get what looks like the wall of a can.

We call this shape a "shell."

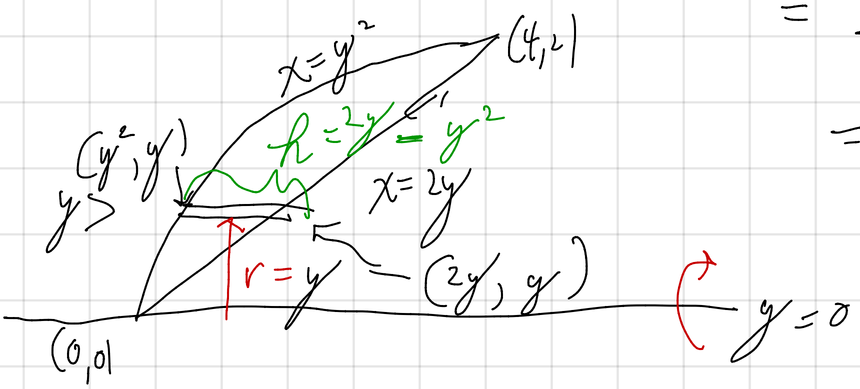
$$2\pi \int_{y=0}^{y=2} r\, h\, dy = 2\pi \int_0^2 y\,(\underbrace{2y}_{\substack{\text{right}\\ x\text{-coord of rec}}} - \underbrace{y^2}_{\substack{\text{left}\\ x\text{-coord of rect}}})\, dy$$

$$= 2\pi \int_0^2 (2y^2 - y^3)\, dy$$

$$= 2\pi \left[\frac{2}{3}y^3 - \frac{1}{4}y^4\right]_0^2$$

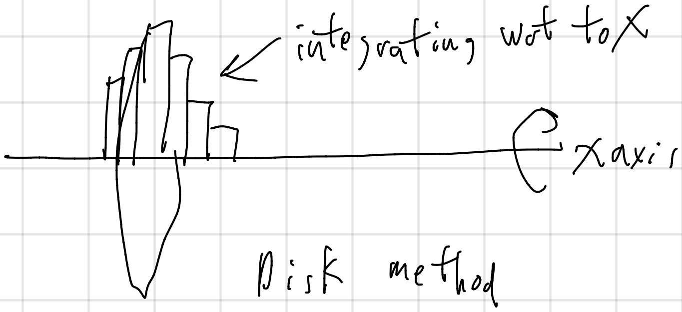
$$= 2\pi \left[\frac{16}{3} - 4\right]$$

$$= 2\pi \left[\frac{16}{3} - \frac{12}{3}\right] = \frac{8}{3}\pi$$

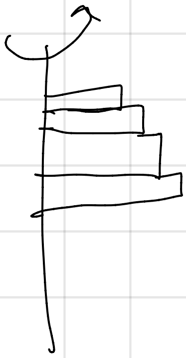


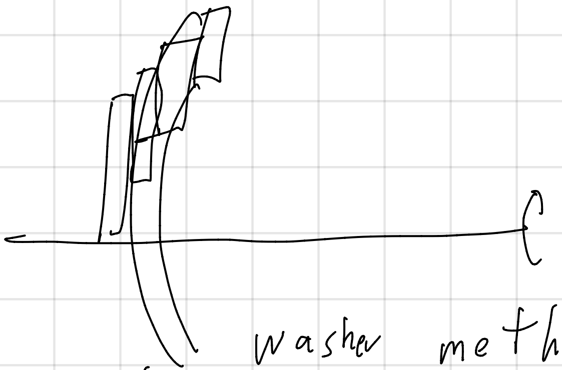
When should I use the shell method?
When should I use the disk/washer method?

If your rectangles are perpendicular to the axis of rotation, use the disk/washer method

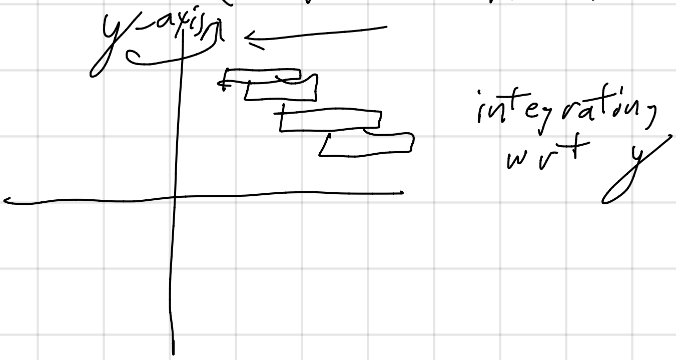


Disk method

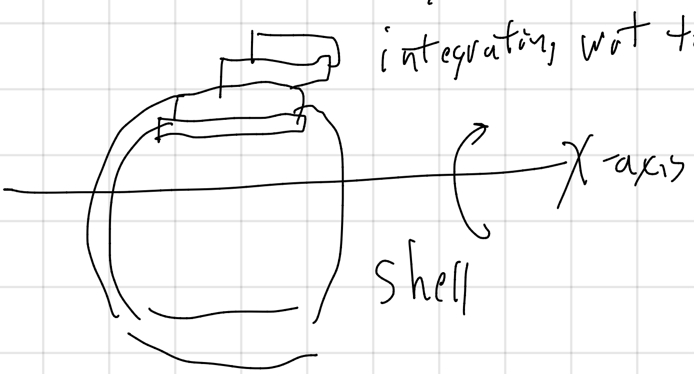




washer method



If your rectangles are parallel to the axis of rotation, then you use the shell method



Shell