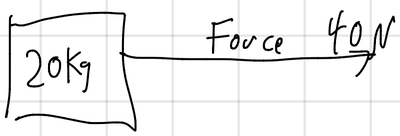
# Math 1b (8:30AM) 29 Jan 2020

## Section 6.4

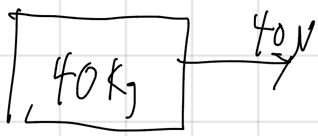
| | MKS | English |
|---|---|---|
| Time | seconds (s) | seconds (s) |
| distance/displacement | meters (m) | feet (ft) |
| velocity | m/s | ft/s |
| acceleration | $m/s^2$ | $ft/s^2$ |
| Mass | Kilograms (Kg) | |
| Force | $Kg\frac{m}{s^2}$ = Netwon (N) | Pounds (lb) |
| Work | | |

$$\text{Force}_{\text{net}} = \text{Mass} \times \text{Acceleration} \qquad \text{Acceleration} = \frac{\text{Force}_{\text{net}}}{\text{Mass}}$$

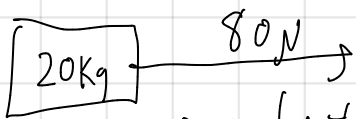
The acceleration of an object is equal to the sum of all the forces acting on that object divided by the mass of that object.




accelerate $2\frac{m}{s^2}$



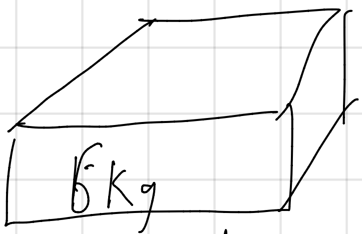

acceleration = $1\frac{m}{s^2}$




accelerate $4\frac{m}{s^2}$

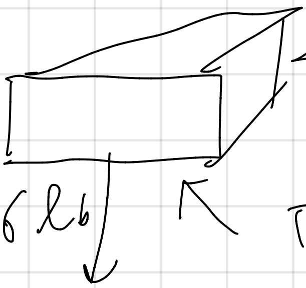
If an object has a mass m, then

the force of gravity on that object (the weight of the object) is $W = mg$ where $g =$ ~~$9.8\frac{m}{s^2}$~~ $10\frac{m}{s^2}$




← Book has a mass of 6 Kg

The weight of the book (the force of gravity on the book) is $(6\text{ Kg})(10\frac{m}{s^2}) = 60\text{ N}$




← Book weighs 6lb

The force of gravity on the book is 6lb.

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What is work?

The work done by a force on an object is (The force) × (The displacement of the object)

Work = Force × displacement

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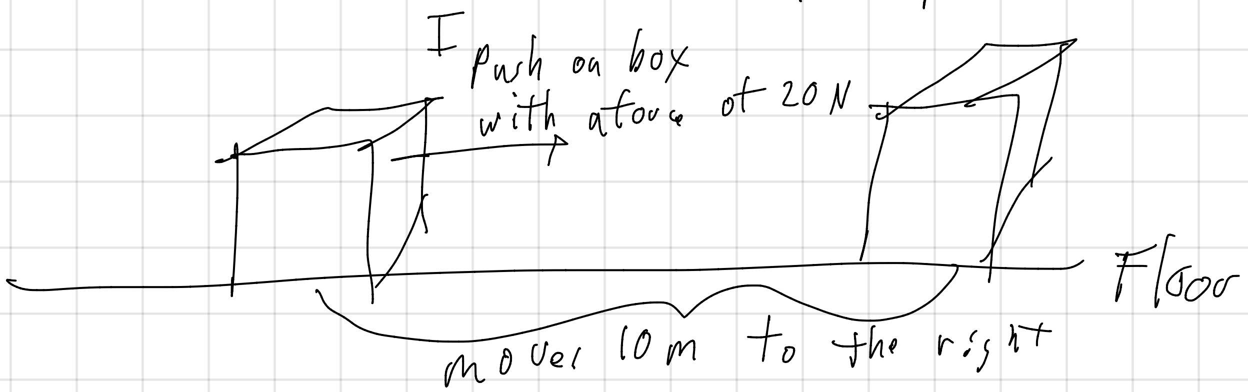



I have done (20 N)(10 m) = 200 Nm of work
= 200 J

| | MKS | English |
|---|---|---|
| displacement | m | ft |
| Force | N | lb |
| Work | Nm = Joules (J) | foot-pound (ft-lb) |

Work done by a force can be positive or negative.

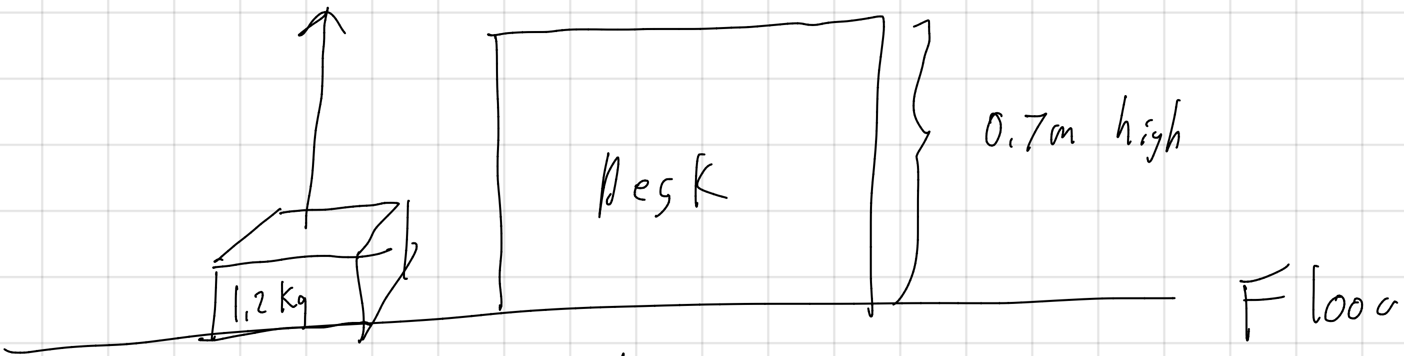
If the force is in the same direction as the displacement, the work done is positive.

If the force is in the opposite direction as the displacement, the work done is negative.

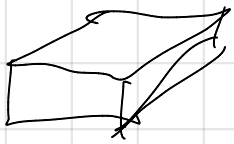
**EXAMPLE 1**

(a) How much work is done in lifting a 1.2-kg book off the floor to put it on a desk that is 0.7 m high? Use the fact that the acceleration due to gravity is $g = 9.8\ \text{m/s}^2$.

(b) How much work is done in lifting a 20-lb weight 6 ft off the ground?




What is the work done by gravity on the book as we lift it 0.7 m?

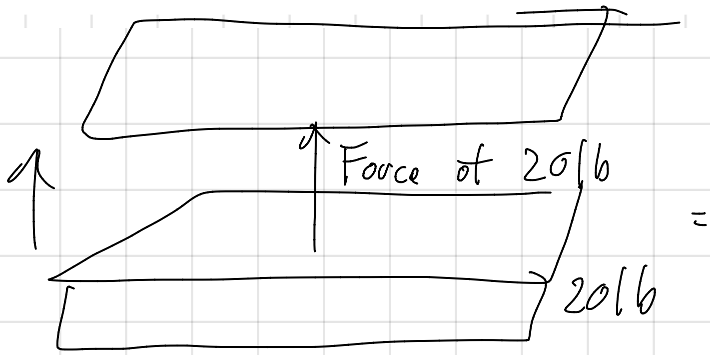


Force of gravity on book = $mg$ = $1.2\ \text{Kg} \cdot (10\ \frac{m}{s^2}) = 12\ N$ (weight of book)

Work done by gravity = $(-12\ N)(0.7m) = -8.4\ J$

Work I must do to lift the box = $(12\ N)(0.7m) = 8.4\ J$

(b) How much work is done in lifting a 20-lb weight 6 ft off the ground?




total work done = $(20\ lb) \cdot (6\ ft)$ = 120 ft-lb

Gravity does −120 ft-lb of work on the book

I must apply 120 ft-lb of work to lift the book

# 6.4 calculus and work

If we apply a constant force F to an object as it moves in one dimension from $x=a$ to $x=b$, ~~it~~ then it is easy to calculate the work done by the force:

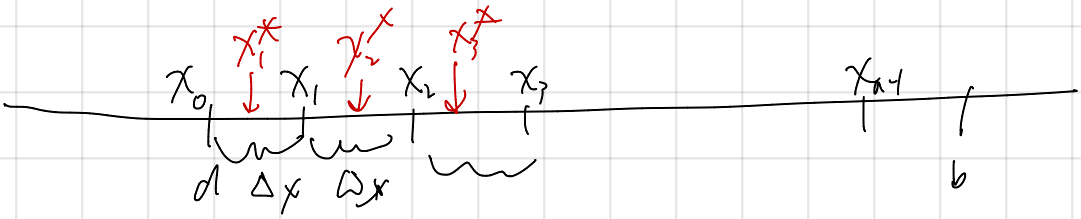
$$\text{Work} = F(b-a)$$

We need calculus to calculate work when the force on the object is not constant.


An object moves from position $x=a$ to $x=b$, A force is applied to the object that depends on the position.

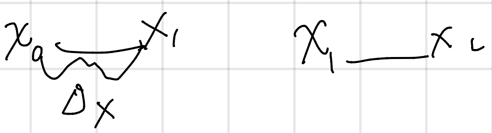
$f(x)$ = force on object at position $x$

How do we find the total work done by $f(x)$ on the object as it moves from $x=a$ to $x=b$?




divide $[a,b]$ into $n$ equal pieces of length $\Delta x = \frac{b-a}{n}$

We estimate the work done by the force $f(x)$ over each sub interval and add them up.




$$f(x_1^*)\Delta x + f(x_2^*)\Delta x_2 + \dots f(x_n^*)\Delta x_n$$

$f(x)$ = force on object at position x

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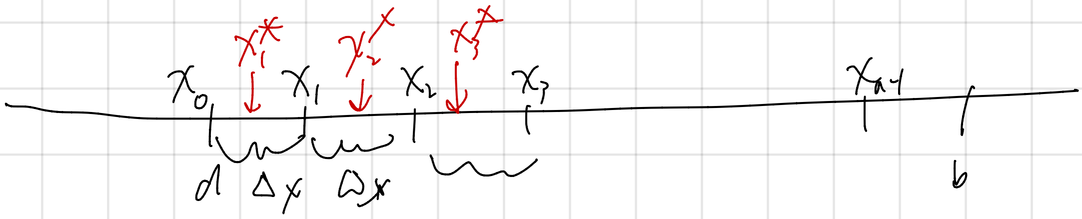


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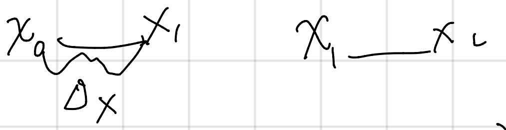


$$\lim_{n\to\infty} \left( f(x_1^*)\Delta x + f(x_2^*)\Delta x_2 + \dots f(x_n^*)\Delta x_n \right)$$

Work done by the force on object $= \int_a^b f(x)\,dx$