Math 1b (8:30AM) 30 Jan 2019

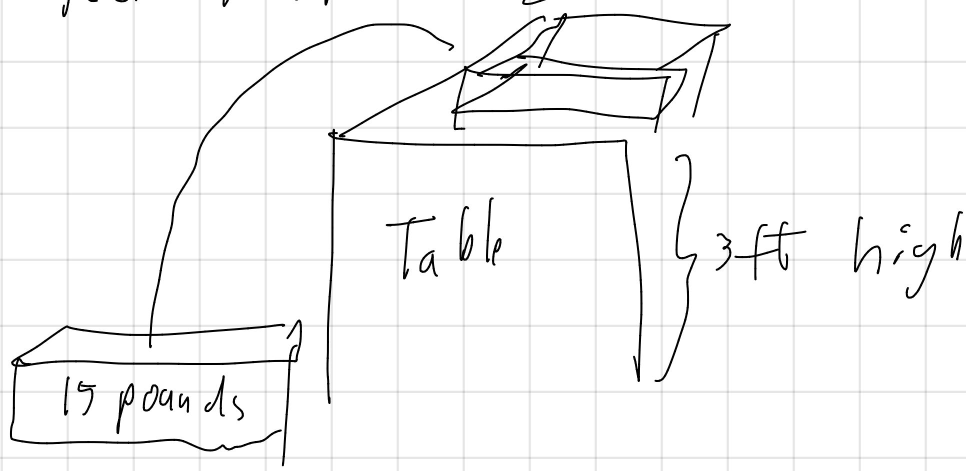
Work = Force x displacement

(The work done on an object by a force)
= (That force) x (The displacement of the object)

The net work on an object changes the speed of the object

If the beginning and ending speeds of an object are zero, the net work on that object must be zero

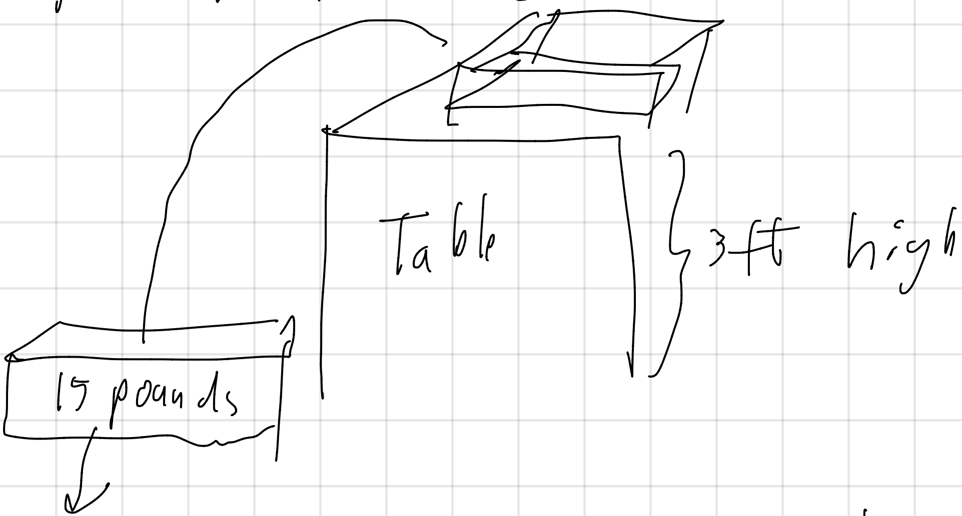
Example



I lift the book & set it on the top of the table

If the beginning and ending speeds of an object are zero, the net work on that object must be zero

Example

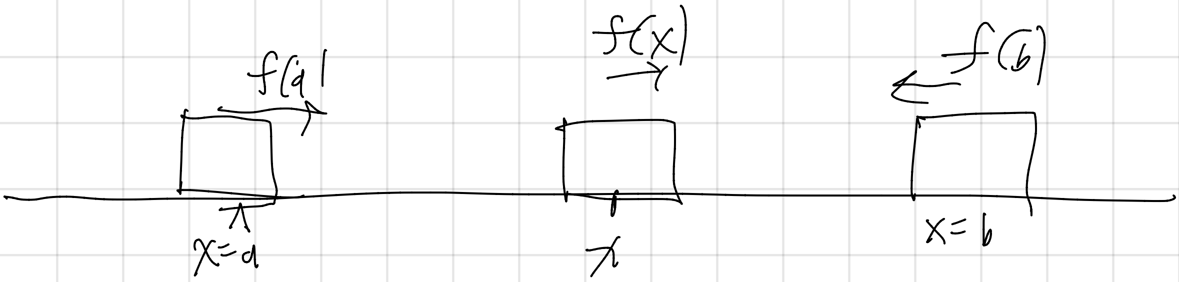


I lift the book & set it on the top of the tabl[e]

since the beginning & ending speed of the book is zero, the net work done on the book must be zero

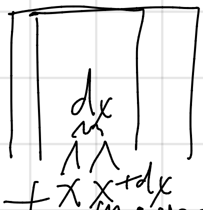
work done by gravity on book = $(-15\text{ lb})(3\text{ ft}) = -45\text{ ft-lb}$
work done by me on book = $(15\text{ lb})(3\text{ ft}) = 45\text{ ft-lb}$

Suppose an object moves from $x=a$ to $x=b$



A varying force is applied to the object

$f(x)$ = Force on the object at position x.



As the object moves from position x to position $x+dx$ (where dx is tiny), the work done on the object over that tiny displacement dx is

$$f(x)\,dx$$

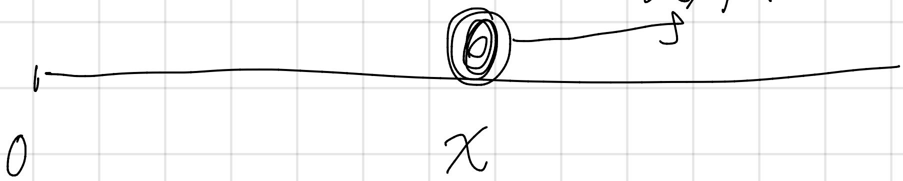
To get the work done over the interval $[a,b]$ we use an integral to add up all those tiny displacements

$$\int_a^b f(x)\,dx$$

EXAMPLE 2 When a particle is located a distance x feet from the origin, a force of $x^2 + 2x$ pounds acts on it. How much work is done in moving it from $x = 1$ to $x = 3$?

by the force $f(x)$

Particle $f(x) = x^2 + 2x$ pounds



$$\int_{x=1}^{x=3} f(x)\,dx = \int_1^3 (x^2+2x)\,dx$$

lb, ft

$$= \left[\frac{1}{3}x^3 + x^2\right]_1^3 = \frac{26}{3} + 8$$

$$= \frac{50}{3} \text{ ft-lb}$$

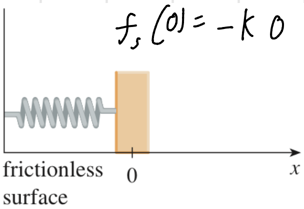
Hooke's constant $k > 0$

$f_s(x)$ = Force on mass due to spring

$f_s(x) = -kx$

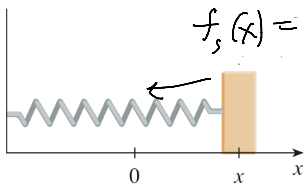
position of mass relative to the neutral position

$f_s(0) = -k0 = 0$



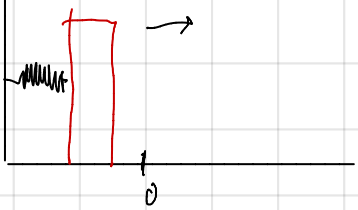
(a) Natural position of spring

$f_s(x) = -kx < 0$



(b) Stretched position of spring

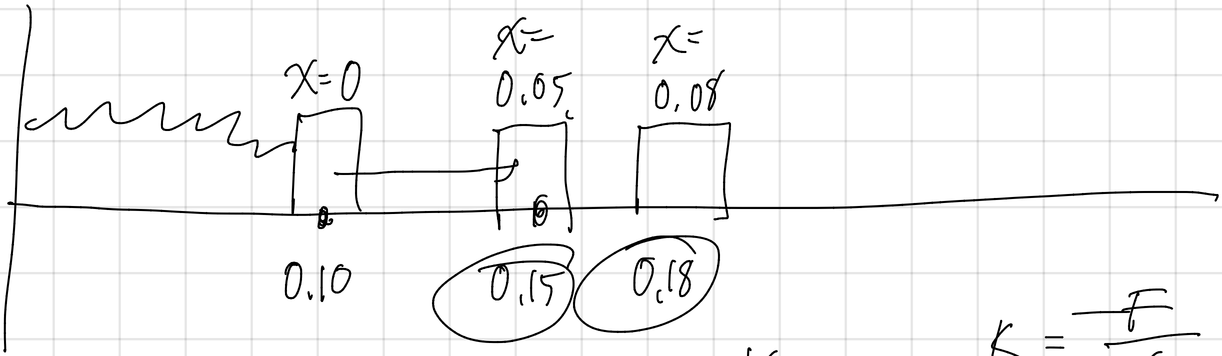
$f_s(x) = -kx > 0$



In the book (Stewart)
$f(x) = kx$
$f(x)$ = The force we must apply to the mass to overcome the spring

EXAMPLE 3 A force of 40 N is required to hold a spring that has been stretched from its natural length of 10 cm to a length of 15 cm. How much work is done in stretching the spring from 15 cm to 18 cm?

0.15m 0.18m



$F = -Kx$ $K = \frac{F}{x}$

$40 = K(0.05)$ $K = \frac{40}{0.05} = 800 \ \frac{N}{m}$

$f(x) = -kx$

$f(x) = kx$

$\int_{0.05}^{0.08} 800x \, dx = 400x^2 \Big|_{0.05}^{0.08}$ Nm or J