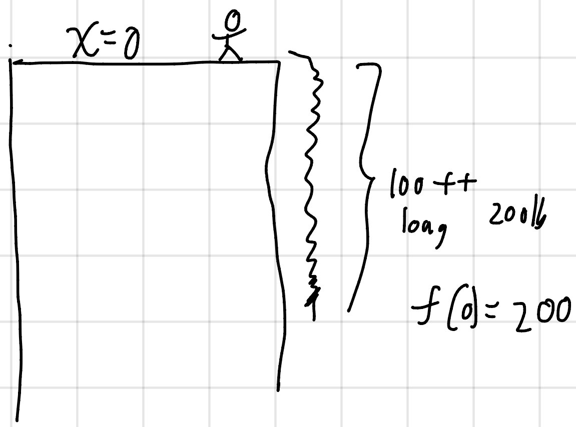
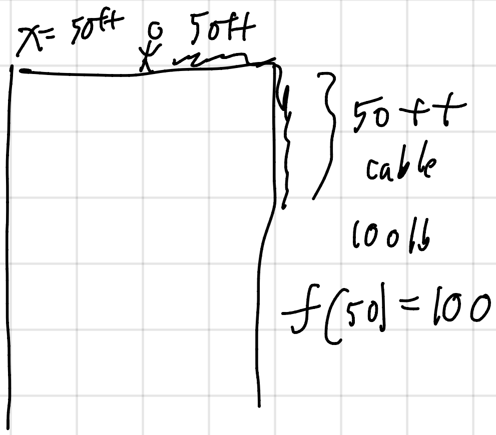
Math 1b (8:30AM) 31 Jan 2020

Take home quiz #3 on web-site
Hand in by Tuesday.

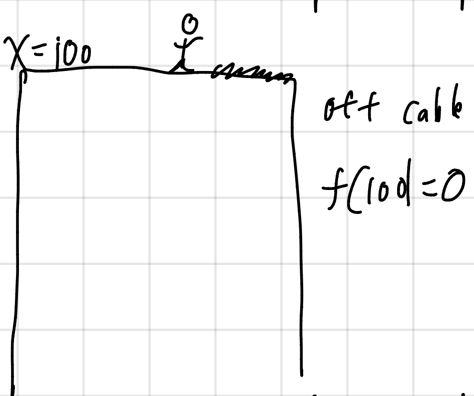
6.4

EXAMPLE 4 A 200-lb cable is 100 ft long and hangs vertically from the top of a tall building. How much work is required to lift the cable to the top of the building?





$x=$ amount of cable pulled up so far (feet)



$f(x)=$ the force the person must apply to pull up the cable when they've pulled up x ft of cable already

weight density of cable $= \frac{200 \text{ lb}}{100 \text{ ft}} = 2 \frac{\text{lb}}{\text{ft}}$

$f(x) = 2(100-x)$

Work done by the person $\displaystyle\int_{x=0}^{x=100} f(x)\,dx = \int_0^{100} 2(100-x)\,dx$

$= \left[200x - x^2\right]_0^{100} = \boxed{10{,}000 \text{ ft-lb}}$

remember to include units in your final answer when doing applied problems.

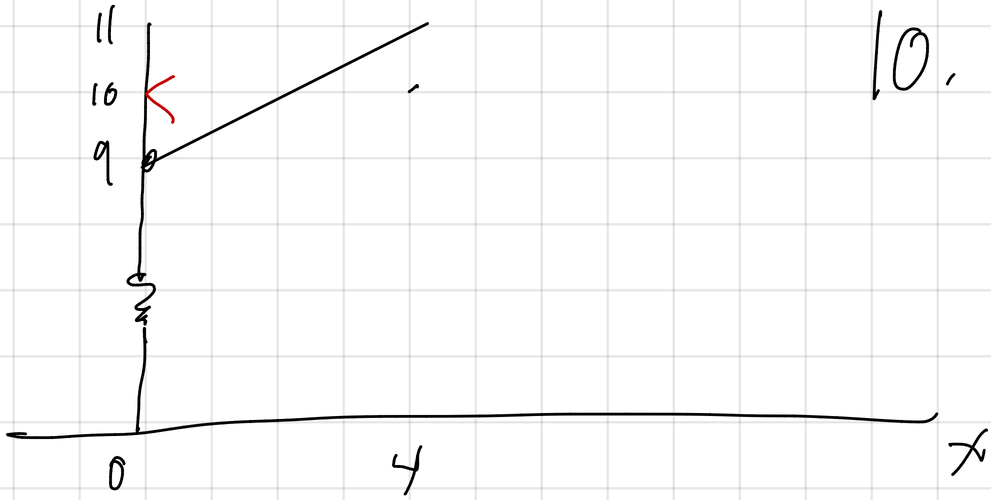
Section 6.5

The average value of a function over an interval.

When we talk about "the average value of a function, we mean the average output value of the function.

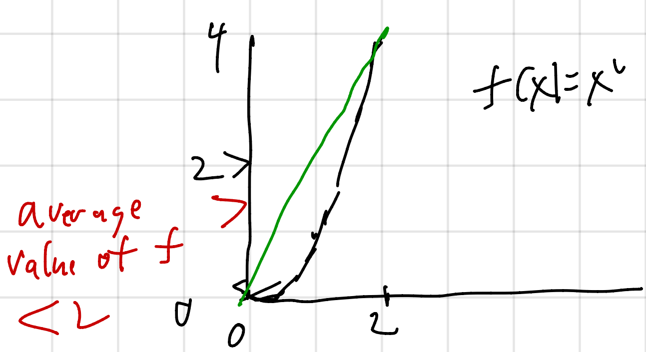
$f(x) = 9 + \frac{1}{2}x$

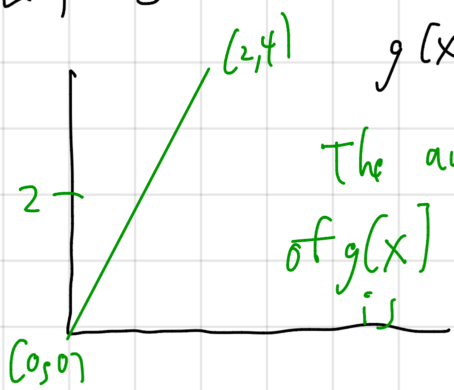
what is the average value of f over $[0, 4]$?



10.

$f(x) = x^2$ what is the averague value of f over $[0, 2]$



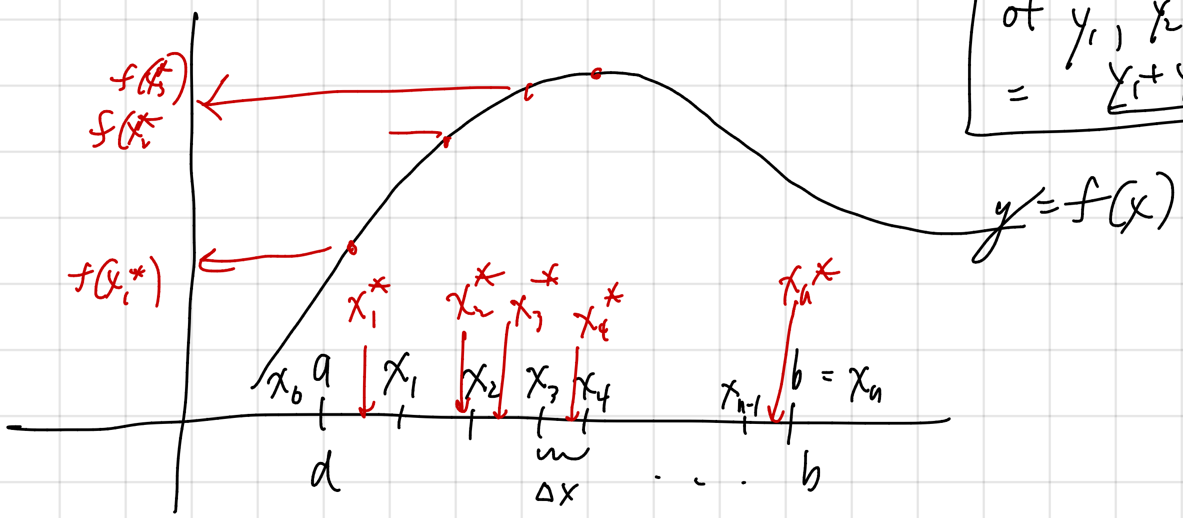


$g(x) = 2x$

The average value of $g(x)$ over $[0,2]$ is 2.

How should we calculate the average value of f over $[a,b]$

Average value of $y_1, y_2, y_3, \dots y_n$
$= \frac{y_1+y_2+y_3+\dots+y_n}{n}$



We divide the interval $[a,b]$ into n equal sub-intervals of length $\Delta x = \frac{b-a}{n}$

We choose a sample point x_k^* from every sub-interval $[x_{k-1}, x_k]$ and we average the function values at those n points

Approximate average value of f over $[a,b]$ $= \frac{f(x_1^*)+f(x_2^*)+f(x_3^*)+\dots+f(x_n^*)}{n}$

Average value of f over $[a,b]$ $= \lim_{n\to\infty} \frac{f(x_1^*)+f(x_2^*)+\dots+f(x_n^*)}{n}$

$\Delta x = \frac{b-a}{n}$

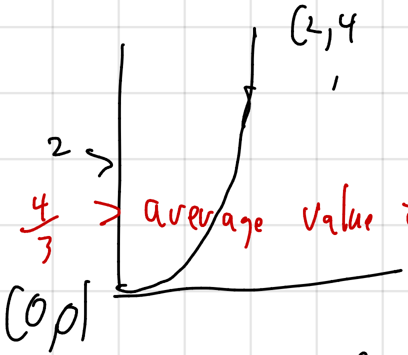
$$= \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*) \frac{1}{n} = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*) \frac{b-a}{n} \cdot \frac{1}{b-a}$$

$$= \frac{1}{b-a} \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*) \Delta x = \frac{1}{b-a} \int_a^b f(x)\,dx$$

$$\boxed{\text{Average value of } f \text{ over } [a,b] = \frac{1}{b-a} \int_a^b f(x)\,dx}$$

$$\text{Average value of } f \text{ over } [a,b] = \frac{1}{b-a}\int_a^b f(x)\,dx$$

What is the average of $f(x) = x^2$ over $[0,2]$?



We expect the average value, f_{ave}

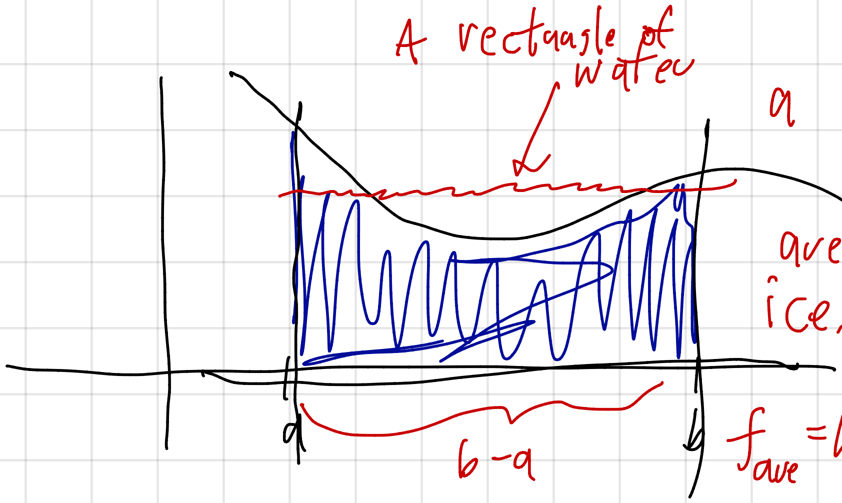
$$0 < f_{ave} < 2$$

$$f_{ave} = \frac{1}{2-0}\int_0^2 x^2\,dx = \frac{1}{2}\cdot\frac{1}{3}x^3\Big|_0^2$$

$$= \frac{4}{3}$$

There is another way to get the formula

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$



The water forms a rectangle whose area is the same as the ice.

$$f_{ave} = \text{height of water} = \frac{\int_a^b f(x)\,dx}{b-a}$$

Suppose I carve a sheet of ice in the shape of my function $0 \le y \le f(x)$, $a \le x \le b$.

The are of my sheet of ice is $\int_a^b f(x)\,dx$

I Know place my sheet of ice between two sheets of glass, one in front, one behind.

I put wooden barriers between the sheets of glass, $y = 0$, $x = a$, and $x = b$

Now I melt the ice, and the barriers and the glass hold the ice in place as it melts.

$f(t)$ = Temperature °F t hours after midnight.

$t=6 \Rightarrow$ 6:00 AM, $t=13 \Rightarrow$ 1:00 PM

What was the average temperature from 8:00 AM to noon?

$$\frac{1}{12-8}\int_{8}^{12} f(t)\,dt \quad °F$$