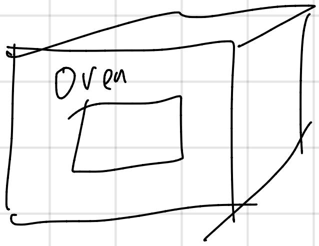
Math 1b (8:30AM) 3 Feb 2020

We heat an oven

$f(t)$ = The temperature of the oven (°F) at time t (min)
$g(t)$ = The rate of increase of temperature of the oven (°F/min) at time t (min)



$$f_{\substack{ave\\[a,b]}} = \frac{1}{b-a}\int_a^b f(t)\,dt \qquad g(t) = f'(t)$$

What is the average temperature of the oven from 20 min to 30 min? (include units in the answer)

$$\frac{1}{30-20}\int_{20}^{30} f(t)\,dt \qquad \underline{°F}$$

What is the average rate of change of the temperature of the oven from 20 to 30 minutes (include units) (There are two differen correct ways of writing the answer)

rate of change of temparature is in units of °F/min and is represented by the function $g(t)$

$$\frac{1}{30-20}\int_{20}^{30} g(t)\,dt \quad °F/min$$

We can also write the average rate of change of the temperature of the oven from 20 to 30 min using $f(t)$ instead of $g(t)$

The average rate of change of temp of oven = The average rate of change of f $= \dfrac{f(30)-f(20)}{30-20}$ $\quad$ °F/min rate of change of temperature is in units of °F/min and is represented by the function $g(t)$

$$\frac{1}{30-20}\int_{20}^{30} g(t)\,dt \quad {}^\circ F/\text{min}$$

We can also write the average rate of change of the temperature of the oven from 20 to 30 min using $f(t)$ instead of $g(t)$

The average rate of change of temp of oven $=$ The average rate of change of f $= \dfrac{f(30)-f(20)}{30-20}$ °F/min

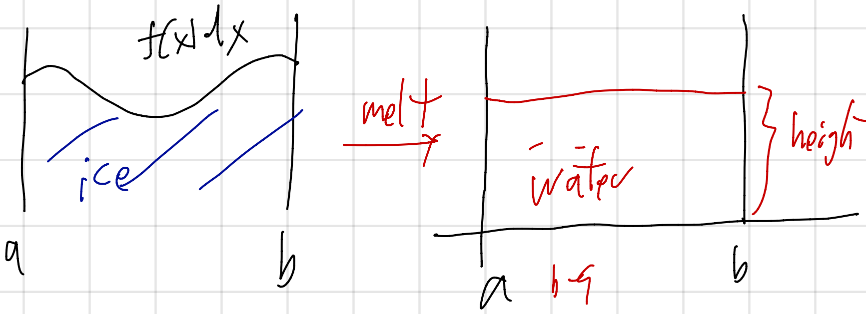
Both of these must give the same answer

$$\frac{1}{30-20}\int_{20}^{30} g(t)\,dt = \frac{f(30)-f(20)}{30-20}$$

$$\int_{20}^{30} g(t)\,dt = f(30)-f(20)$$

(Fundamental Theorem of Calculus II)

The average value of f over an interval $[a,b]$ is $\dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$



height $= \dfrac{\int_a^b f(x)\,dx}{b-a}$

$=$ average value of f

$f(x) = 7$ what is the average value of f over $[0, 100]$

$$\cancel{\int_0^{100} f(x)\,dx = 700} \; ?$$

$$\frac{1}{100}\int_0^{100} f(x)\,dx = \frac{1}{100}\cdot 700 = 7$$

Chapter 7, sections 7.1-7.5 (material for midterm 3)

7.1-7.5 covers techiques for integration

The fundamental theorem of calculus II (5.3)
Integration by substion (5.5)
Integration by parts (7.1)
Techniques for integrating trig function (7.2)
Integration using trigonometric substitution (7.3)
Integration using partial fractions (7.4)
Review of all the techniques 7.1-7.4 (7.5)

Section 7.1: Integration by Parts

Integration by parts comes from the product rule for differentiation

$$\frac{d}{dx} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

$$\int f(x)g(x)\,dx = ?$$

Assume f is differentiable, and assume $G(x)$ is an antiderivative of $g(x)$. $G'(x) = g(x)$.

$$\frac{d}{dx} f(x)G(x) = f'(x)G(x) + \underline{f(x)\overset{G'(x)}{g(x)}}$$

$$\int \left(f'(x)G(x) + \underline{f(x)g(x)}\right)dx = f(x)G(x) + c$$

$$\underline{\int f'(x)G(x)\,dx} + \int f(x)g(x)\,dx = f(x)G(x) + c$$

$$\boxed{\int f(x)g(x)\,dx = f(x)G(x) - \int f'(x)G(x)\,dx}$$

Integration by parts

Integration by parts using the table method

	diff	int
+	$f(x)$	$g(x)$
−	$f'(x)$	$G(x)$
+		

$$\int f(x)g(x)\,dx = f(x)G(x) - \int f'(x)G(x)\,dx$$

$$\boxed{\int f(x)\,g(x)\,dx = f(x)\,G(x) - \int f'(x)\,G(x)\,dx}$$

Integration by parts

Integration by parts using the table method

	diff	int
+	$f(x)$	$g(x)$
−	$f'(x)$	$G(x)$
+		

$$\int f(x)\,g(x)\,dx = f(x)\,G(x) - \int f'(x)\,G(x)\,dx$$

$$\int x \cos x \, dx$$

	diff	int
+	x	$\cos x$
−	1	$\sin x$
+		

$$\int x \cos x\, dx = x \sin x - \int \sin x \, dx$$

$$= x \sin x - (-\cos x) + C$$

$$= \boxed{x \sin x + \cos x + C}$$

Check our answer: $\frac{d}{dx}(x \sin x + \cos x) =$

$$= 1 \cdot \sin x + x \cos x - \sin x = x \cos x$$

	diff	int
+	$\cos x$	x
−	$-\sin x$	$\frac{1}{2}x^2$
+		

$$\int x \cos x\, dx = \frac{1}{2}x^2 \cos x + \frac{1}{2}\int x^2 \sin x \, dx$$

True, but does not help