Math 1b (8:30AM) 4 Feb 2020

Integration by Parts

$$\int x \cos x \, dx$$

$$= x \sin x - \int \sin x \, dx$$

$$= \boxed{x \sin x + \cos x + c}$$

	diff	int
$+$	x	$\cos x$
$-$	1	$\sin x$

$$\boxed{\int f(x) g(x) \, dx = f(x)G(x) - \int f'(x) G(x) \, dx}$$

Integration by Part

$$\int x^2 \cos x \, dx$$

$$= x^2 \sin x + 2x \cos x - 2 \int \cos x \, dx$$

$$= x^2 \sin x + 2x \cos x - 2 \sin x + c$$

	diff	int
$+$	x^2	$\cos x$
$-$	$2x$	$\sin x$
$+$	2	$-\cos x$

Check answer:

$$\frac{d}{dx}\left(x^2 \sin x + 2x \cos x - 2 \sin x\right)$$

$$\cancel{2x \sin x} + x^2 \cos x + \cancel{2 \cos x} - \cancel{2x \sin x} - \cancel{2 \cos x} = x^2 \cos x$$

$$\int x^2 \cos x \, dx \qquad \int 0 \, dx$$

$$= \boxed{x^2 \sin x + 2x \cos x - 2 \sin x + c}$$

	diff	int
$+$	x^2	$\cos x$
$-$	$2x$	$\sin x$
$+$	2	$-\cos x$
$-$	0	$-\sin x$

$$\int x^2 \cos x \, dx$$

$$\int 0 \, dx$$

$$= \boxed{x^2 \sin x + 2x \cos x - 2\sin x + C}$$

	diff	int
$+$	x^2	$\cos x$
$-$	$2x$	$\sin x$
$+$	2	$-\cos x$
$-$	0	$-\sin x$

	diff	int
$+$	x^2	$\cos x$
$-$	$2x$	$\sin x$
$+$		

	~~diff~~	int
$-$	$2x$	$\sin x$
$+$	2	$-\cos x$

There are three different kinds of integration by parts probems: Type I, Type II, Type III

Type I

$$\int (\text{polynomial})(\cos x \text{ or } \sin x \text{ or } e^x)$$

diff	int
polynomial	cos x or e^x or sin x
↓	cos x, e^x or sin x
↓	cos x, e^x or sin x
↓	cos x, e^x or sin x

differentiate until 0

Type II integration by parts

$$\int \ln x \, dx$$

$$= \int 1 \cdot \ln x \, dx$$

$$= x \ln x - \int \underset{x \cdot \frac{1}{x}}{1} \, dx$$

$$= \boxed{x \ln x - x + C}$$

	diff	int
+	$\ln x$	1
−	$\frac{1}{x}$	x

check answer: $\frac{d}{dx}(x \ln x - x)$

$$= 1 \cdot \ln x + x \cdot \frac{1}{x} - 1 = \ln x + 1 - 1 = \ln x \; \checkmark$$

$$\int \frac{\ln x}{x^2}\,dx$$

	dif	int
$+$	$\ln x$	$\frac{1}{x^2} = x^{-2}$
$-$	$\frac{1}{x}$	$-x^{-1}$

$$\int \frac{\ln x}{x^2}\,dx = -x^{-1}\ln x + \int \frac{1}{x}\cdot\frac{1}{x}\,dx$$

$$= -\frac{\ln x}{x} + \int x^{-2}\,dx$$

$$= -\frac{\ln x}{x} - \frac{1}{x} + C$$

constant, $\alpha \neq -1$

$$\int x^{\alpha} \ln x\,dx$$

	diff	int
$+$	$\ln x$	x^{α}
$-$	$\frac{1}{x}$	$\frac{1}{1+\alpha}x^{\alpha+1}$ (assume $\alpha \neq -1$)

$$= \frac{x^{\alpha+1}}{1+\alpha}\ln x - \frac{1}{1+\alpha}\int x^{\alpha}\,dx$$

$$= \frac{x^{\alpha+1}}{1+\alpha}\ln x - \frac{1}{(1+\alpha)^2}x^{\alpha+1} + C$$

Type II integration by parts

$$\int \begin{pmatrix} 1 \text{ or} \\ x^{\alpha} \end{pmatrix} \cdot \left(\ln x \text{ or } \arctan x \text{ or } \arcsin x \text{ or } (\ln x)^2 \text{ or } (\ln x)^3\right)$$

Type III integration by Parts

$$\int e^x \cos x \, dx$$

	diff	int
+	e^x	$\cos x$
−	e^x	$\sin x$
+	e^x	$-\cos x$

It does not matter which of the e^x and $\cos x$ I differentiate integrate

$$\int e^x \cos x \, dx = e^x \sin x + e^x \cos x - \int e^x \cos x \, dx$$

$$+\int e^x \cos x \, dx \qquad\qquad +\int e^x \cos x \, dx$$

$$2\int e^x \cos x \, dx = e^x \sin x + e^x \cos x + C$$

$$\int e^x \cos x \, dx = \frac{1}{2}\left(e^x \sin x + e^x \cos x\right) + C$$

$$\boxed{\int e^x \cos x \, dx = \frac{1}{2} e^x (\sin x + \cos x) + C}$$

check answer: $\frac{d}{dx}\left(\frac{1}{2} e^x (\sin x + \cos x)\right)$

$$= \frac{1}{2} e^x (\sin x + \cos x) + \frac{1}{2} e^x (\cos x - \sin x)$$

$$= \frac{1}{2} e^x (\cancel{\sin x} + \cos x + \cos x - \cancel{\sin x})$$

$$= \frac{1}{2} e^x (2\cos x) = e^x \cos x \quad \checkmark$$