Math 1b (8:30AM) 7 Feb 2020

Ⓐ Integrate $\int \sin^3\theta \cos^2\theta \, d\theta$

Ⓑ Integrate $\int \sin^4\theta \cos^3\theta \, d\theta$

Ⓒ Rewrite $\int \sin^4\theta \cos^2\theta \, d\theta$ as a sum of integrals of even powers of cosine

$$u = \cos\theta \qquad -du = \sin\theta \, d\theta$$

$$\int \sin^3\theta \cos^2\theta \, d\theta = \int \sin^2\theta \cos^2\theta \underbrace{\sin\theta \, d\theta}_{-du}$$

$$= \int (1 - \underbrace{\cos^2\theta}_{u}) \underbrace{\cos^2\theta}_{u} \underbrace{\sin\theta \, d\theta}_{-du} = -\int (1-u^2) u^2 \, du$$

$$u = \sin\theta \qquad du = \cos\theta \, d\theta$$

$$\int \sin^4\theta \cos^3\theta \, d\theta = \int \sin^4\theta \cos^2\theta \underbrace{\cos\theta \, d\theta}_{du}$$

$$= \int \sin^4\theta (1 - \sin^2\theta) \cos\theta \, d\theta = \int u^4 (1-u^2) \, du$$

$$\int \sin^4\theta \cos^2\theta \, d\theta = \int (\sin^2\theta)^2 \cos^2\theta \, d\theta$$

$$= \int (1-\cos^2\theta)^2 \cos^2\theta \, d\theta$$

$$= \int (1 - 2\cos^2\theta + \cos^4\theta) \cos^2\theta \, d\theta = \int \cos^2\theta \, d\theta - 2\int \cos^4\theta \, d\theta + \int \cos^6\theta \, d\theta$$

How do we integrate $\int \cos^n\theta \, d\theta$ when n is even?

Two possible Answers

- Use trignometric identities

$$\cos^2\theta = \frac{1}{2}\cos 2\theta + \frac{1}{2}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

Two possible Answers

- Use trignometric identities

$$\cos^2\theta = \frac{1}{2}\cos 2\theta + \frac{1}{2}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

- A more efficent method is integration by parts

$$\int \cos^2\theta \, d\theta$$

	dif	int
$+$	$\cos\theta$	$\cos\theta$
$-$	$-\sin\theta$	$\sin\theta$

$$\int \cos^2\theta \, d\theta = \cos\theta\sin\theta + \int \sin^2\theta \, d\theta$$

$$\int \cos^2\theta \, d\theta = \cos\theta\sin\theta + \int (1-\cos^2\theta) \, d\theta$$

$$\int \cos^2\theta \, d\theta = \int \cos^2\theta \, d\theta = \cos\theta\sin\theta + \int 1 \, d\theta - \int \cos^2\theta \, d\theta$$

$$+ \int \cos^2\theta \, d\theta \qquad\qquad + \int \cos^2\theta \, d\theta$$

$$2\int \cos^2\theta \, d\theta = \cos\theta\sin\theta + \theta + c$$

$$\boxed{\int \cos^2\theta \, d\theta = \frac{1}{2}\cos\theta\sin\theta + \frac{1}{2}\theta}$$

	dif	int
$+$	$\cos^{n-1}\theta$	$\cos\theta$
$-$	$(n-1)\cos^{n-2}\theta(-\sin\theta)$	$\sin\theta$

$$\int \cos^n\theta \, d\theta = \int \cos^{n-1}\theta \cos\theta \, d\theta$$

$$\int \cos^n\theta \, d\theta = \sin\theta\cos^{n-1}\theta + (n-1)\int \sin^2\theta \cos^{n-2}\theta \, d\theta$$

$$\int \cos^n\theta \, d\theta = \sin\theta\cos^{n-1}\theta + (n-1)\int (1-\cos^2\theta)\cos^{n-2}\theta \, d\theta$$

$$1\cdot\int \cos^n\theta \, d\theta = \sin\theta\cos^{n-1}\theta + (n-1)\int \cos^{n-2}\theta \, d\theta - (n-1)\int \cos^n\theta \, d\theta$$

$$+(n-1)\int \cos^n\theta \, d\theta \qquad\qquad +(n-1)\int \cos^n\theta \, d\theta$$

$$n\int \cos^n\theta \, d\theta = \sin\theta\cos^{n-1}\theta + (n-1)\int \cos^{n-2}\theta \, d\theta$$

$$\boxed{\int \cos^n\theta \, d\theta = \frac{1}{n}\sin\theta\cos^{n-1}\theta + \frac{n-1}{n}\int \cos^{n-2}\theta \, d\theta}$$

"Reduction Formula"

$$\int \overbrace{\cos^0}^{1}\theta \, d\theta = \theta + C$$

$$\int \cos^0 d\theta$$

$$\underline{\int \cos^2\theta \, d\theta} = \frac{1}{2}\sin\theta\cos\theta + \frac{1}{2}(\theta) + C$$

$$\int \cos^4\theta \, d\theta = \frac{1}{4}\sin\theta\cos^3\theta + \frac{3}{4}\left(\frac{1}{2}\sin\theta\cos\theta + \frac{1}{2}\theta\right) + C$$

$$= \frac{1}{4}\sin\theta\cos^3\theta + \frac{3}{8}\sin\theta\cos\theta + \frac{3}{8}\theta + C$$

$$\int \cos^6\theta \, d\theta = \ldots$$

$$\int \cos^8\theta \, d\theta = \ldots$$

$$\int \sec^m\theta \tan^n\theta \, d\theta$$

$$\int \tan\theta \, d\theta \qquad\qquad \int \sec\theta \, d\theta$$

$$= \int \frac{\sin\theta}{\cos\theta} \, d\theta \qquad u = \cos\theta$$

$$du = -\sin\theta \, d\theta$$

$$-du = \sin\theta \, d\theta$$

$$= -\int \frac{1}{u} \, du$$

$$= -\ln|u| + c$$

$$= -\ln|\cos\theta| + c$$

$$= \ln\left|\frac{1}{\cos\theta}\right| + c = \ln|\sec\theta| + c$$

$$-\ln|A| = \ln\left|\frac{1}{A}\right|$$

$$\boxed{\int \tan x \, dx = \ln|\sec\theta| + C}$$

Memorize

$$\int \sec\theta \, d\theta$$

In section 7.4, I will show you how to derive this

$$\int \sec\theta \, d\theta = \ln|\sec\theta + \tan\theta| + c$$

$$\frac{d}{d\theta} \ln|\sec\theta + \tan\theta| = \frac{\sec\theta\tan\theta + \sec^2\theta}{\sec\theta + \tan\theta} = \frac{\boxed{\sec\theta}\cancel{(\tan\theta + \sec\theta)}}{\cancel{\sec\theta + \tan\theta}}$$