Math 1b (8:30AM)

10 Feb 2020

$- x^n \begin{cases} (\ln x)^2 \\ \arctan x \\ \arcsin x \end{cases}$

$$\int (\ln x)^2 \, dx$$

Type II

	dif	int
$+$	$(\ln x)^2$	1
$-$	$\frac{2 \ln x}{x}$	x

$$\int (\ln x)^2 \, dx = x (\ln x)^2 - \int 2 \ln x \, dx$$

	dift	int
$+$	$-2 \ln x$	1
$-$	$-2/x$	x

$$= x (\ln x)^2 - 2x \ln x + \int 2 \, dx$$

$$= \boxed{x (\ln x)^2 - 2x \ln x + 2x + C}$$

$$\int \sin^m x \cos^n x \, dx$$

if sin is raised to an odd power,

let $u = \cos x$ $\quad du = -\sin x \, dx$

$$= -\int \underbrace{\sin^{m-1} x} \cos^n x \underbrace{\sin x \, dx}_{du}$$

$$= -\int \left(1-\cos^2 x\right)^{\frac{m-1}{2}} \cos^n x \sin x \, dx = -\int \left(1-u^2\right)^{\frac{m-1}{2}} u^n \, du$$

if cos is raised to an odd power

let $u = \sin x$ $\quad du = \cos x \, dx$

if cos & sin are both raised to an even power, convert everything to cosines, and use trig identities, or integration by parts.

$\int \sec^m x \tan^n x \, dx$

$$= \int \underbrace{\sec^{m-2} x}\, \overbrace{\tan^n x}^{u^n}\, \overbrace{\sec^2 x \, dx}^{du}$$ Let $u = \tan x$, $du = \sec^2 x dx$

$$= \int (1+\tan^2 x)^{\frac{m-2}{2}} \tan^n x \sec^2 x dx$$

$$= \int (1+u^2)^{\frac{m-2}{2}} u^n \, du$$

This will only work if sec is raised to an <u>even</u> power

$\int \sec^m x \tan^n x \, dx$

when sec is raised to an even power, use $u = \tan x dx$, $du = \sec^2 x dx$

Example

secant is raised to an even power

$$\int \sec^6 x \tan^6 x dx$$

$$= \int \underline{\sec^4 x}\, \overbrace{\tan^6 x}^{u^6}\, \overbrace{\sec^2 x \, dx}^{du}$$ $u = \tan x$ $du = \underline{\sec^2 x dx}$

$$= \int (1+\tan^2 x)^2 \tan^6 x \sec^2 x dx$$

$$= \int (1+u^2)^2 u^6 du$$

$$\int \sec^m x \tan^n x \, dx$$

$u = \sec x$

$du = \underline{\sec x \tan x \, dx}$

$$= \int \overset{u^{m-1}}{\sec^{m-1} x} \tan^{n-1} x \overbrace{(\sec x \tan x \, dx)}^{du}$$

$$= \int \sec^{m-1} x \, (\sec^2 x - 1)^{\frac{n-1}{2}} \sec x \tan x \, dx$$

$$= \int u^{m-1} (u^2 - 1)^{\frac{n-1}{2}} du$$

This will work if $n-1$ is even, if n is odd if tan is originally raised to an odd power.

If sec is raised to an even power,

$u = \tan x$

$du = \sec^2 x \, dx$

If tan is raised to an odd power we

$u = \sec x$

$du = \sec x \tan x$

$$\int \sec^m x \tan^n x \, d$$

Example

$$\int \sec^5 x \tan^5 x \, dx$$

$u = \sec x$

$du = \sec x \tan x \, dx$

$$= \int \overset{u^4}{\sec^4 x} \tan^4 x \overbrace{\sec x \tan x \, dx}^{du}$$

$$= \int \sec^4 x \, (\sec^2 x - 1)^2 \sec x \tan x \, dx = \int u^4 (u^2 - 1)^2 du$$

The difficult case is when sec is raised to an odd power and tan is raised to an even power.

Example

$$\int \sec x \tan^2 x \, dx$$

step #1

The difficult case is when sec is raised to an odd power and tan is raised to an even power.

Example

$$\int \sec x \tan^2 x \, dx$$

step #1

convert everything into integrals of odd powers of sec.

$$\int \sec x \tan^2 x \, dx = \int \sec x (\sec^2 x - 1) \, dx$$

$$= \int \sec^3 x \, dx - \int \sec x \, dx$$

We memorize $\int \sec x \, dx = \ln|\sec x + \tan x| + C$

For higher odd powers of sec, we use integration by parts.

	diff	int
$+$	$\sec x$	$\sec^2 x$
$-$	$\sec x \tan x$	$\tan x$

$$\int \sec^3 x \, dx = \sec x \tan x - \int \sec x \tan^2 x \, dx$$

$$\int \sec^3 x \, dx = \sec x \tan x - \int \sec x (\sec^2 x - 1) \, dx$$

$$\int \sec^3 x \, dx = \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx$$

$$+ \int \sec^3 x \, dx \qquad\qquad + \int \sec^3 x \, dx$$

$$2 \int \sec^3 x \, dx = \sec x \tan x + \ln|\sec x + \tan x| + c$$

$$\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + c$$

$$\int \sec x \tan^2 x \, dx = \int \sec x (\sec^2 x - 1) \, dx = \int \sec^3 x \, dx - \underline{\int \sec x \, dx}$$

$$= \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| - \ln|\sec x + \tan x| = \frac{1}{2} \sec x \tan x - \frac{1}{2} \ln|\sec x + \tan x|$$

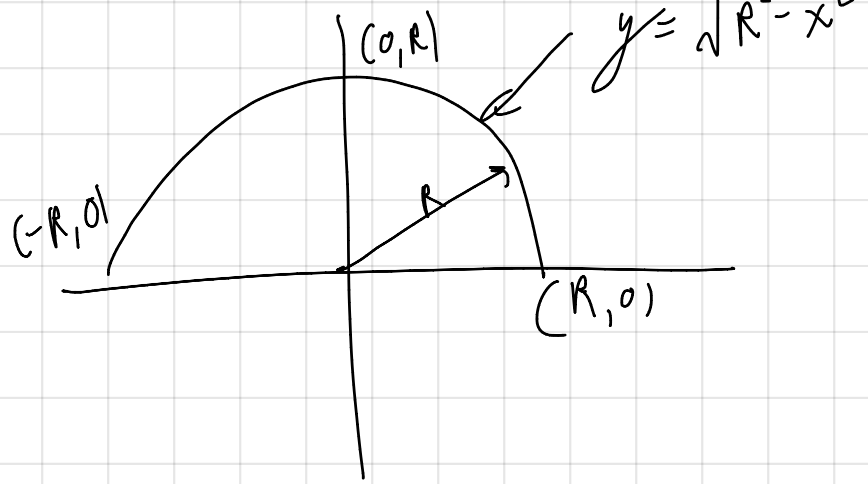
$$\boxed{\int \sec^3 x\,dx = \frac{1}{2}\sec x\tan x + \frac{1}{2}\ln|\sec x+\tan x| + C}$$

$$\int \sec x\tan^2 x\,dx = \int \sec x(\sec^2 x - 1)\,dx = \int \sec^3 x\,dx - \underline{\int \sec x\,dx}$$

$$= \frac{1}{2}\sec x\tan x + \frac{1}{2}\ln|\sec x+\tan x| - \ln|\sec x+\tan x| + C = \frac{1}{2}\sec x\tan x - \frac{1}{2}\ln|\sec x+\tan x| + C$$

Section 7.3

Find the area of a circle of radius R (R is a constant).



$x^2+y^2=R^2$

$y^2=R^2-x^2$

$y=\sqrt{R^2-x^2}$

semi circle

Area of a circle of radius R $= 2\int_{-R}^{R}\sqrt{R^2-x^2}\,dx$

x	$\theta=\arcsin\frac{x}{R}$
$-R$	$-\pi/2$
R	$\pi/2$

$\theta = \arcsin\frac{x}{R}$ $\left(-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}\right)$

$\sin\theta = x/R$

$x = R\sin\theta$ $\quad dx = R\cos\theta\,d\theta$

$$2\int_{-R}^{R}\sqrt{R^2-x^2}\,dx$$

$$= 2\int_{-\pi/2}^{\pi/2} R\cos\theta\; R\cos\theta\,d\theta$$

$$\sqrt{R^2-x^2} = \sqrt{R^2-(R\sin\theta)^2}$$

$$= 2R^2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^2\theta\,d\theta$$

$\cos\frac{\pi}{2} = \cos\frac{-\pi}{2} = 0$

$$= \sqrt{R^2(1-\sin^2\theta)} = \sqrt{R^2\cos^2\theta} = R\cos\theta$$

$$= \cancel{2}R^2\left[\cancel{\frac{1}{2}}\sin\theta\,\cancel{\cos\theta} + \cancel{\frac{1}{2}}\theta\right]_{-\pi/2}^{\pi/2} = R^2\left(\frac{\pi}{2}-\left(\frac{-\pi}{2}\right)\right) = \pi R^2$$