Math 1b (8:30 AM) 18 Feb 2020

$$\int \sec x \, dx = ?$$

$$\int \frac{1}{\cos x} dx = \int \cos^{-1} x \, dx$$

when integrating $\int \sin^m x \cos^{odd} x \, dx$ we let

$u = \underline{\sin x}$ $du = \underline{\cos x \, dx}$

$$\int \sin^m x \overbrace{\cos^{odd-1} x}\ \underbrace{\cos x \, dx}$$

$$= \int \overset{u^m}{\sin^m x} (\overset{1-u^2}{1-\sin^2 x})^{\frac{odd-1}{2}} du$$

$u = \sin x$

$du = \cos x \, dx$

$$\int \frac{1}{\cos x} dx = \int \frac{1}{\cos^2 x} \overbrace{\cos x \, dx}$$

$$= \int \frac{1}{1-\sin^2 x} \overbrace{\cos x \, dx}^{du} = \int \frac{1}{1-u^2} du$$

$$\frac{1}{1-u^2} = \frac{A}{1-u} + \frac{B}{1+u} \quad (1-u)^2 \quad = \boxed{\frac{1/2}{1-u} + \frac{1/2}{1+u}}$$

$$1 = A(1+u) + B(1-u)$$

$$1 = A + Au + B - Bu$$

$$1 = \overset{A-B=0}{(A-B)u} + \overset{A+B=1}{(A+B)} \qquad A = B = \frac{1}{2}$$

$$u = \sin x$$

$$du = \cos x\,dx$$

$$\int \frac{1}{\cos x}\,dx = \int \frac{1}{\cos^2 x}\,\overbrace{\cos x\,dx}$$

$$= \int \frac{1}{1-\sin^2 x}\,\overbrace{\cos x\,dx}^{du} = \int \frac{1}{1-u^2}\,du$$

$$\frac{1}{1-u^2} = \frac{A}{1-u} + \frac{B}{1+u} \quad (1-u)^2 \quad = \boxed{\frac{1/2}{1-u} + \frac{1/2}{1+u}}$$

$$1 = A(1+u) + B(1-u)$$

$$1 = A + Au + B - Bu$$

$$1 = \underbrace{(A-B)}_{A-B=0}u + \underbrace{(A+B)}_{A+B=1} \qquad A = B = \frac{1}{2}$$

$$\int \frac{1}{1-u^2}\,du = \frac{1}{2}\int \frac{1}{1+u}\,du + \frac{1}{2}\int \frac{1}{1-u}\,du$$

$$= \frac{1}{2}\ln|1+u| - \frac{1}{2}\ln|1-u|$$

$$= \frac{1}{2}\ln\left|\frac{1+u}{1-u}\right| = \frac{1}{2}\ln\left|\frac{1+\sin\theta}{1-\sin\theta}\right| + C$$

$$= \frac{1}{2}\ln\left|\frac{1+\sin\theta}{1-\sin\theta}\cdot\frac{1+\sin\theta}{1+\sin\theta}\right| + C$$

$$= \frac{1}{2}\ln\left|\frac{(1+\sin\theta)^2}{1-\sin^2\theta}\right| + C = \frac{1}{2}\ln\left|\frac{(1+\sin\theta)^2}{\cos^2\theta}\right| + C$$

$$= \ln\sqrt{\left|\frac{(1+\sin\theta)^2}{\cos\theta}\right|} + C = \ln\left|\frac{1+\sin\theta}{\cos\theta}\right| + C = \ln|\sec\theta + \tan\theta| + C$$

$$\int \frac{2x+5}{(x-3)^2}\,dx$$

proper rational function, no long division required

write template

$$\frac{2x+5}{(x-3)^2} = \frac{A}{x-3} + \frac{B}{(x-3)^2}$$

linear factor (squared)

linear factor must appear to first and second powers

$$(x-3)^2\left(\frac{2x+5}{(x-3)^2}\right) = \left(\frac{A}{x-3} + \frac{B}{(x-3)^2}\right)(x-3)^2$$

$$2x+5 = A(x-3)+B$$

$$2x+5 = Ax-3A+B$$

$$2x+5 = (A)x + (-3A+B)$$

$A=2$ $\qquad -3A+B=5$

$-6+B=5$

$B=11$

Check

$$\frac{2(x-3)}{(x-3)^2} + \frac{11}{(x-3)^2} = \frac{2x-6+11}{(x-3)^2}$$ ✓

$$\int \frac{2x+5}{(x-3)^2}\,dx = \int \frac{2}{x-3}\,dx + \int \frac{11}{(x-3)^2}\,dx$$

$$\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = -x^{-1}+c = \frac{-1}{x}+c$$

$$= 2\ln|x-3| - \frac{11}{x-3} + C$$

If $\int f(x)\,dx = F(x)+c$

then $\int f(mx+k)\,dx = \frac{1}{m}F(mx+k)+c$

$$\int \frac{x^2 - x + 1}{x^3 + 2x^2 + 2x}\,dx$$

proper rational function, no long division needed,

$$\left(\frac{x^2-x+1}{x\,\underbrace{(x^2+2x+2)}_{\text{irreducible quadratic}}} = \frac{A}{x} + \frac{Bx+C}{x^2+2x+2}\right) x\,(x^2+2x+2)$$

factor denominator

$$x^2-x+1 = A(x^2+2x+2) + (Bx+C)x$$

$$x^2 - x + 1 = Ax^2 + 2Ax + 2A + Bx^2 + Cx$$

$$x^2 - x + 1 = (A+B)x^2 + (2A+C)x + 2A$$

$$A + B = 1 \qquad 2A + C = -1 \qquad 2A = 1$$

$$A = \frac{1}{2}$$

$$B = \frac{1}{2}$$

$$C = -2$$

$$\int \frac{x^2-x+1}{x(x^2+2x+2)}\,dx = \int \frac{\frac{1}{2}}{x}\,dx + \int \frac{\frac{1}{2}x-2}{x^2+2x+2}\,dx$$

$$= \frac{1}{2}\ln|x| + \int \frac{\frac{1}{2}x-2}{(x+1)^2+1}\,dx \qquad u = x+1 \quad du = dx$$

$$x = u-1$$

$$= \frac{1}{2}\ln|x| + \int \frac{\frac{1}{2}u - \frac{5}{2}}{u^2+1}\,du = \frac{1}{2}\ln|x| + \frac{1}{4}\int \frac{2u}{u^2+1}\,du - \frac{5}{2}\int \frac{1}{u^2+1}\,du$$

$$= \frac{1}{2}\ln|x| + \frac{1}{4}\ln\big(\overbrace{x^2+2x+2}^{u^2+1}\big) - \frac{5}{2}\arctan\big(\overbrace{x+1}^{u}\big) + C$$