Math 1b (8:30AM) 19 Feb 2020

~~5.4~~

Rational substitution,

$u = \sqrt{x}$, $\quad u = \sqrt[3]{x} \quad u = \sqrt{x+4}$

$$u = \sqrt{\sqrt{x}+1}$$

$$u = \sqrt{x} \xrightarrow{\text{Normal}} \cancel{du = \frac{1}{2\sqrt{x}}dx}$$

$$u = \sqrt{x} \longrightarrow \begin{array}{l} x = u^2 \\ dx = 2u\,du \end{array}$$

Example

$$\int e^{\sqrt{x}}dx \qquad u = \sqrt{x} \qquad du = \frac{1}{2\sqrt{x}}dx$$

I'm stuck. There is no $\frac{1}{\sqrt{x}}dx$ in the problem to substitute for.

$$\int e^{\sqrt{x}}dx \qquad u = \sqrt{x} \qquad x = u^2 \quad dx = 2u\,du$$

$$= \int e^u\, 2u\, du$$

Example

$\int e^{\sqrt{x}}\,dx \qquad u=\sqrt{x} \qquad du=\frac{1}{2\sqrt{x}}\,dx$

I'm stuck. There is no $\frac{1}{\sqrt{x}}\,dx$ in the problem to substitute for.

$\int e^{\sqrt{x}}\,dx \qquad u=\sqrt{x} \qquad x=u^2 \qquad dx=2u\,du$

$= \int e^u\, 2u\, du$

$= \int 2u\, e^u\, du$

$= 2u\,e^u - 2e^u + C$

$= 2\sqrt{x}\,e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$

Type I

	diff	int
$+$	$2u$	e^u
$-$	2	e^u
$+$	0	e^u

$$\int \sqrt{1+\sqrt{x}}\,dx$$

$$= \int u \, (4u^3 - 4u)\,du$$

$$= \int (4u^4 - 4u^2)\,du$$

$$= \frac{4}{5}u^5 - \frac{4}{3}u^3 + C$$

$$= \frac{4}{5}\sqrt{1+\sqrt{x}}^{\,5} - \frac{4}{3}\sqrt{1+\sqrt{x}}^{\,3} + C$$

$u = \sqrt{1+\sqrt{x}}$

$\cancel{du = \frac{1}{2}}$

$\sqrt{1+\sqrt{x}} = u$

$1+\sqrt{x} = u^2$

$\sqrt{x} = u^2 - 1$

$x = (u^2-1)^2$

$dx = 2(u^2-1)\,2u\,du$

$dx = (4u^3 - 4u)\,du$

This is similar to trig-substitutions

$u = \sqrt{x}$	$\theta = \arcsin x$
$x = u^2$	$x = \sin\theta$
$dx = 2u\,du$	$dx = \cos\theta\,d\theta$
	$\sqrt{1-x^2} = \cos\theta$

Section 5.2

The comparison properties for integrals

If (1) $a < b$

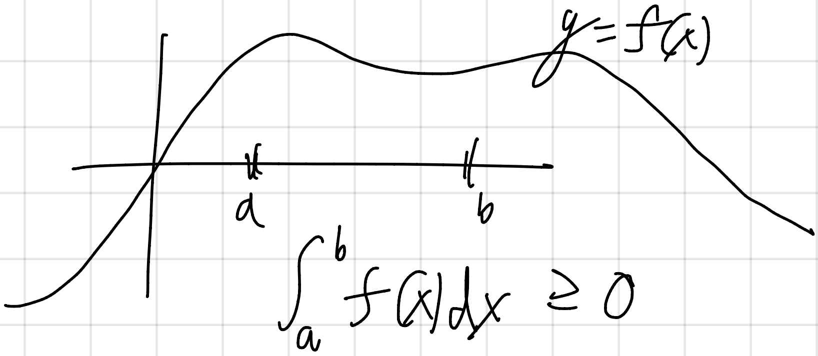
(2) $f(x) \geq 0$ for all x, $a \leq x \leq b$

(3) $\int_a^b f(x)\,dx$ exists

Then $$\int_a^b f(x)\,dx \geq 0$$

The integral of a non-negative function is non-negative.

$$\int_a^b f(x)\,dx$$



$$\int_a^b f(x)\,dx \geq 0$$

Suppose (1)-(3) are true above.

Then $$\int_a^b f(x)\,dx = \lim_{n\to\infty} \underbrace{\sum_{k=1}^{n} f(x_k^*)\,\Delta x}_{\text{This is } \geq 0 \text{ for all } n}$$

so this limit is ≥ 0.

If $A_n \geq 0$ for all n,
and if $\lim_{n\to\infty} A_n$ exists, then $\lim_{n\to\infty} A_n \geq 0$

5.2

Comparison Properties of the Integral

6. If $f(x) \geqslant 0$ for $a \leqslant x \leqslant b$, then $\int_a^b f(x)\,dx \geqslant 0$.

7. If $f(x) \geqslant g(x)$ for $a \leqslant x \leqslant b$, then $\int_a^b f(x)\,dx \geqslant \int_a^b g(x)\,dx$.

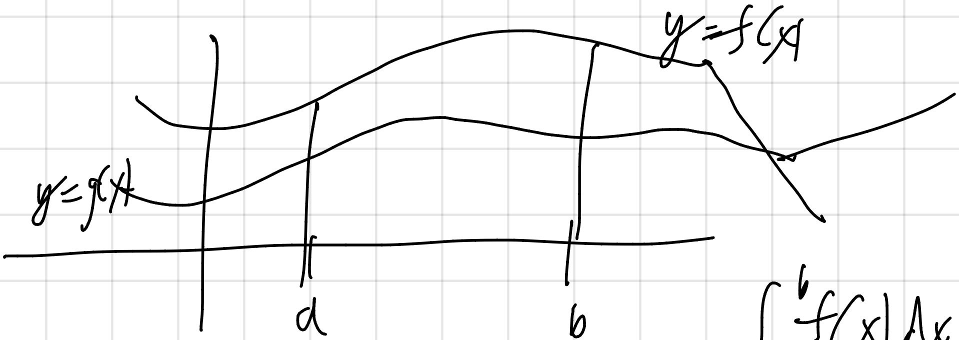
8. If $m \leqslant f(x) \leqslant M$ for $a \leqslant x \leqslant b$, then

$$m(b-a) \leqslant \int_a^b f(x)\,dx \leqslant M(b-a)$$

7 If $a<b$, $f(x) \geq g(x)$ for $a \leq x \leq b$, $\int_a^b f(x)dx$ and $\int_a^b g(x)dx$ are integrable, then

$$\int_a^b f(x)dx \geq \int_a^b g(x)dx$$

"A bigger function has a bigger integral."



$$\int_a^b f(x)dx \geq \int_a^b g(x)dx$$

Proof $\int_a^b f(x)dx - \int_a^b g(x)dx$

$$= \int_a^b \underbrace{(f(x)-g(x))}_{\substack{f(x) \geq g(x) \\ f(x)-g(x) \geq 0}} dx \geq 0$$

$$\int_a^b f(x)dx - \int_a^b g(x)dx \geq 0 \qquad \int_a^b f(x)dx \geq \int_a^b g(x)dx$$

$\square$

Use the comparison properties of integrals to give an upper and lower estimate for

$$\int_0^1 \frac{1}{1+x^3}\,dx$$

$$\int_0^1 \frac{1}{1+x^2}\,dx = \arctan x \Big|_0^1 = \frac{\pi}{4}$$

compare $\frac{1}{1+x^3}$ to $\frac{1}{1+x}$?

for $\boxed{0 < x < 1}$

$x^2 < x$

$x^3 < x^2$

$1+x^3 < 1+x^2$

$\frac{1}{1+x^3} > \frac{1}{1+x^2}$

$$\int_0^1 \frac{1}{1+x^3}\,dx \geq \int_0^1 \frac{1}{1+x^2}\,dx$$

$$\int_0^1 \frac{1}{1+x^3} \geq \frac{\pi}{4} \approx 0.75$$

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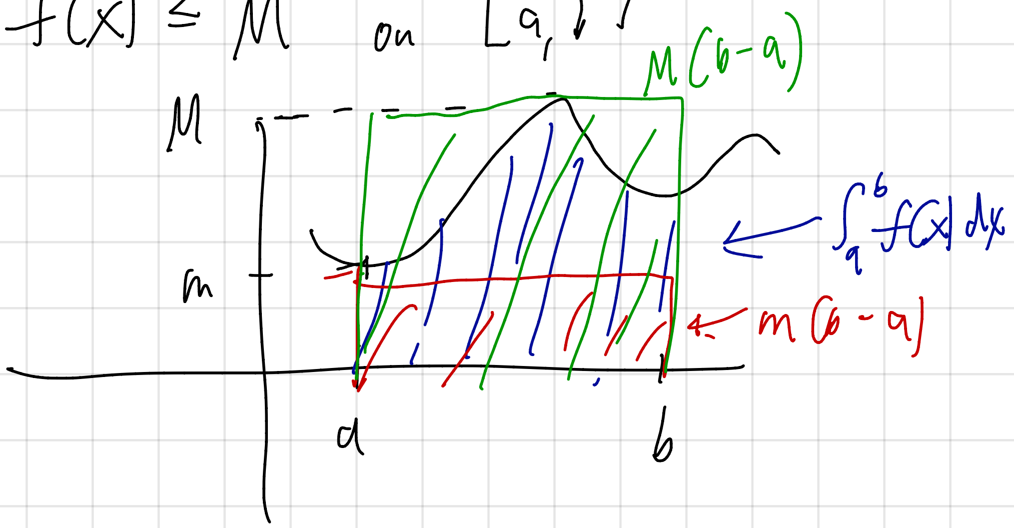
for $0 \leq x \leq 1$ $\boxed{\frac{1}{1+x^3} \leq 1}$

$$\int_0^1 \frac{1}{1+x^3}\,dx \leq \int_0^1 1\,dx = 1$$

$$0.75 \leq \frac{\pi}{4} \leq \boxed{\int_0^1 \frac{1}{1+x^3}\,dx} \leq 1$$

Suppose that $f(x)$ is a function $[a, b]$,
$f(x)$ is integrable on $[a, b]$,

$$m \le f(x) \le M \quad \text{on } [a, b]$$



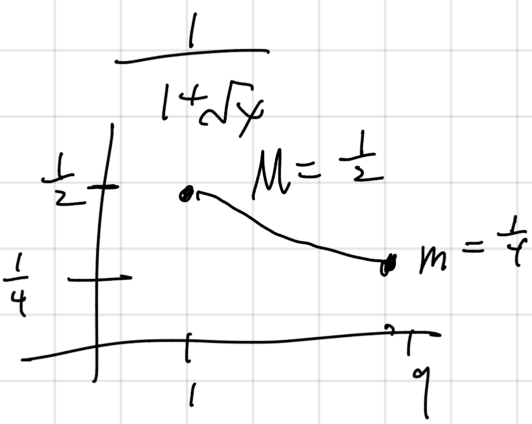
$$\int_a^b m\,dx \le \int_a^b f(x)\,dx \le \int_a^b M\,dx$$

$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$

Estimate $\displaystyle\int_1^9 \frac{1}{1+\sqrt{x}}\,dx$

$$\frac{1}{4}(9-1) \le \int_1^9 \frac{1}{1+\sqrt{x}}\,dx \le \frac{1}{2}(9-1)$$

$$2 \le \int_1^9 \frac{1}{1+\sqrt{x}}\,dx \le 4$$



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