Math 1b (8:30 AM) 21 Feb 2020

5.2

$$\textbf{28.} \int \frac{x^3+6x-2}{x^4+6x^2}\,dx$$

Proper function, no long division required

$$x^2(x^2+6)\left(\frac{x^3+6x-2}{x^2(x^2+6)}\right) = \left(\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+6}\right)x^2(x^2+6)$$

$$x^3+0x^2+6x-2 = Ax(x^2+6) + B(x^2+6) + (Cx+D)x^2$$

$x=0$: $\quad -2 = 0 + B(6) + 0$

$B = -1/3$

$$x^3+0x^2+6x-2 = (A+C)x^3 + (B+D)x^2 + (6A)x + (6B)$$

$A+C=1 \qquad -\frac{1}{3}+D=0 \qquad 6A=6 \qquad (6\cdot\frac{-1}{3})=-2$ ✓

$D=\frac{1}{3} \qquad A=1$

$C=0$

$$\int\frac{x^3+6x-2}{x^2(x^2+6)} = \int\frac{1}{x}\,dx + \int\frac{-1/3}{x^2}\,dx + \int\frac{1/3}{x^2+6}\,dx$$

$$= \ln|x| + \frac{1}{3x} + \frac{1}{3\sqrt{6}}\arctan\frac{x}{\sqrt{6}} + c$$

$$\boxed{\int\frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + c}$$

memorizing

$$= \frac{1}{a^2}\int\frac{1}{\left(\frac{x}{a}\right)^2+1}\,dx = \frac{1}{a}\int\frac{1}{a}\cdot\frac{1}{\left(\frac{x}{a}\right)^2+1}\,dx = \frac{1}{a}\arctan\frac{x}{a} + c$$

Comparison Properties of the Integral

$a < b$

6. If $f(x) \geqslant 0$ for $a \leqslant x \leqslant b$, then $\int_a^b f(x)\,dx \geqslant 0$.

The integral of a ~~positive~~ non-negative function is positive

7. If $f(x) \geqslant g(x)$ for $a \leqslant x \leqslant b$, then $\int_a^b f(x)\,dx \geqslant \int_a^b g(x)\,dx$.

8. If $m \leqslant f(x) \leqslant M$ for $a \leqslant x \leqslant b$, then

$$m(b-a) \leqslant \int_a^b f(x)\,dx \leqslant M(b-a)$$