Math 1b (8:30AM) 27 Feb 2020

Before we can use the comparison theorem, we need a library of integrals that we already know converge or diverge

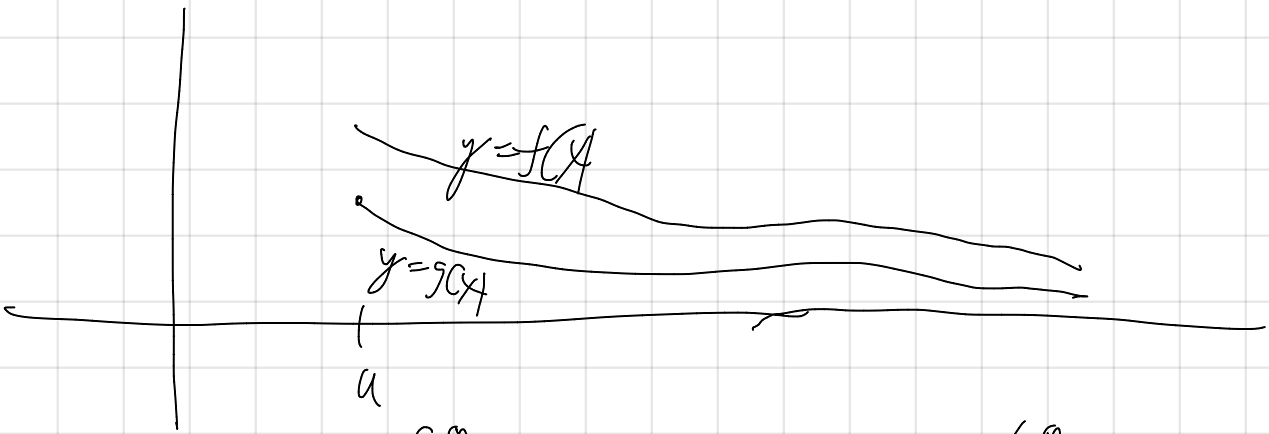
converge		diverge	
$\int_1^\infty \frac{1}{x^p}\,dx$, $p>1$	$\int_0^1 \frac{1}{x^p}\,dx$, $p<1$	$\int_1^\infty \frac{1}{x^p}\,dx$, $p\le 1$	$\int_0^1 \frac{1}{x^p}\,dx$, $p\ge 1$
$\int_1^\infty \frac{1}{x^3}\,dx$	$\int_0^1 \frac{1}{\sqrt{x}}\,dx$	$\int_1^\infty \frac{1}{\sqrt{x}}\,dx$	$\int_0^1 \frac{1}{x^3}\,dx$
$\int_0^\infty e^{-x}\,dx = \int_0^\infty \frac{1}{e^x}\,dx$		$\int_1^\infty \frac{5}{\sqrt{x}}\,dx$	$\int_0^{0.01} \frac{1}{x^3}\,dx$

$$\underbrace{\int_0^\infty 17e^{-x}\,dx \qquad \int_{-1000}^\infty 17e^{-x}\,dx}_{\text{converges}}$$

Comparison Theorem Suppose that f and g are continuous functions with $f(x) \geqslant g(x) \geqslant 0$ for $x \geqslant a$.

(a) If $\int_a^\infty f(x)\,dx$ is convergent, then $\int_a^\infty g(x)\,dx$ is convergent.

(b) If $\int_a^\infty g(x)\,dx$ is divergent, then $\int_a^\infty f(x)\,dx$ is divergent.



If $\int_a^\infty f(x)\,dx$ converges then $\int_a^\infty g(x)\,dx$ also converges

$$0 \leq \int_a^\infty g(x)\,dx \leq \int_a^\infty f(x)\,dx$$

To show $\int_a^\infty g(x)\,dx$ converges using the comparison test, find a bigger function $f(x)$, $f(x) \geq g(x) \geq 0$ for $x \geq a$. such that $\int_a^\infty f(x)\,dx$ converges,

Example

state whether $\int_1^\infty \frac{1}{1+x^3}\,dx$ converges or diverges, Justify your answer.

$1+x^3 > x^3 > 0$

If $x \geq 1$, then $0 < \frac{1}{1+x^3} < \frac{1}{x^3}$

Example

State whether $\int_1^\infty \frac{1}{1+x^3}\,dx$ converges or diverges. Justify your answer.

$1+x^3 > x^3 > 0$

$\boxed{\text{If } x \geq 1, \text{ then } 0 \leq \frac{1}{1+x^3} \leq \frac{1}{x^3}}$

$\boxed{\text{and } \int_1^\infty \frac{1}{x^3}\,dx \text{ converges}}$

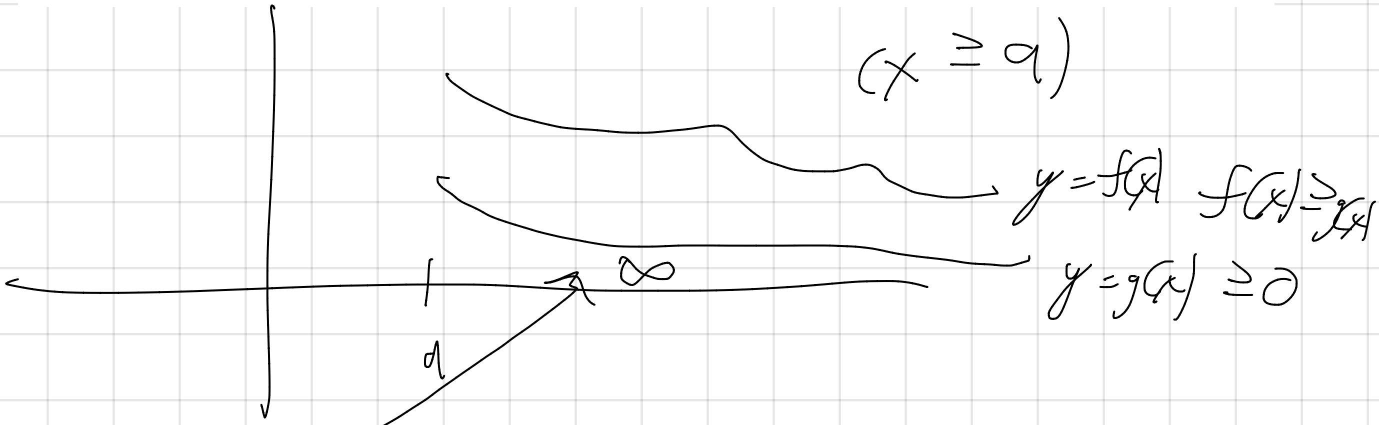
Therefore by the comparison test $\boxed{\int_1^\infty \frac{1}{1+x^3}\,dx \text{ converges}}$

Write these words to get full credit

Comparison Theorem Suppose that f and g are continuous functions with $f(x) \geqslant g(x) \geqslant 0$ for $x \geqslant a$.

(a) If $\int_a^\infty f(x)\,dx$ is convergent, then $\int_a^\infty g(x)\,dx$ is convergent.

(b) If $\int_a^\infty g(x)\,dx$ is divergent, then $\int_a^\infty f(x)\,dx$ is divergent.



If $\int_a^\infty g(x)\,dx$ diverges so does $\int_a^\infty f(x)\,dx$

To use the comparison test to show $\int_a^\infty f(x)\,dx$ diverges, find a smaller function $g(x) \leq f(x)$ such that $\int_a^\infty g(x)\,dx$ diverges.

Example

Use the comparison test to determine whether

$$\int_1^\infty \frac{2+\sin x}{x}\,dx$$

converges or diverges.

If $x \geq 1$, $\quad -1 \leq \sin x \leq 1$

Example

Use the comparison test to determine whether $\int_1^\infty \frac{2+\sin x}{x}\,dx$ converges or diverges.

If $x \geq 1$,

$$-1 \leq \sin x \leq 1$$

$$+2 \quad\quad +2 \quad\quad +2$$

$$1 \leq 2+\sin x \leq 3$$

$$\frac{1}{x} \leq \frac{2+\sin x}{x} \leq \frac{3}{x}$$

For $x \geq 1$, $\frac{2+\sin x}{x} \geq \frac{1}{x} \geq 0$ and $\int_1^\infty \frac{1}{x}\,dx$ diverges.

Therefore by the comparison theorem $\int \frac{2+\sin x}{x}\,dx$

Write these words

also diverges.

$$\int_a^\infty h(x)\,dx \qquad h(x) \geq 0 \quad \text{for } x \geq a$$

To use the comparison theorem to show $\int_a^\infty h(x)\,dx$ converges

find a bigger function, $f(x)$ $0 \leq h(x) \leq f(x)$

whose integral converges $\int_a^\infty f(x)\,dx$ converges