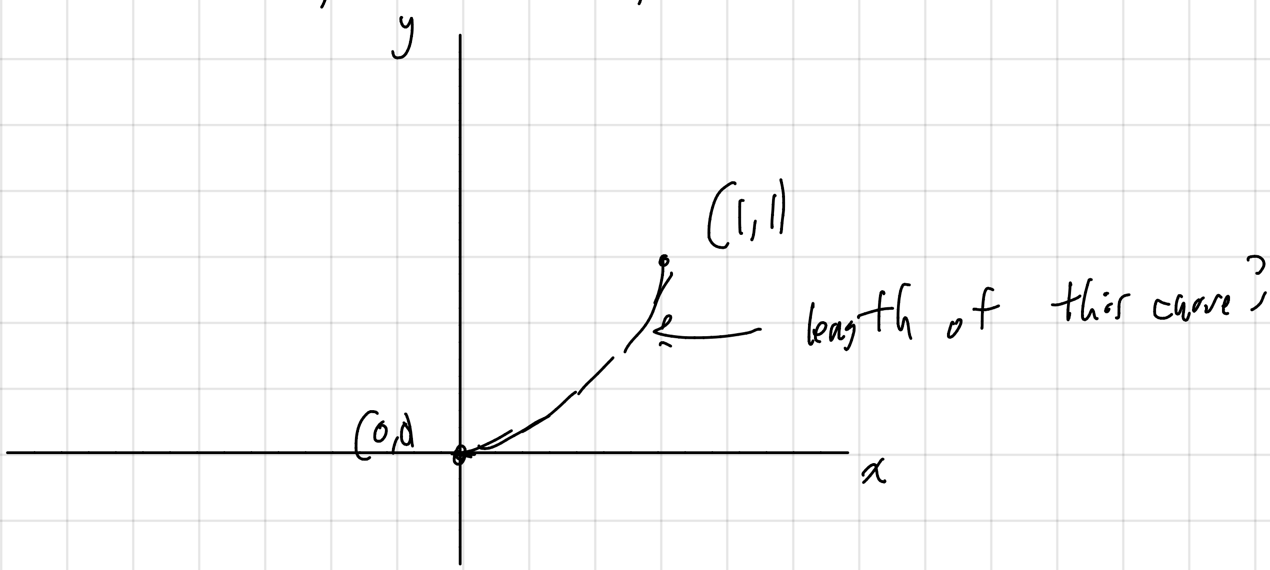
Math 1b (8:30 AM)   28 Feb 2020

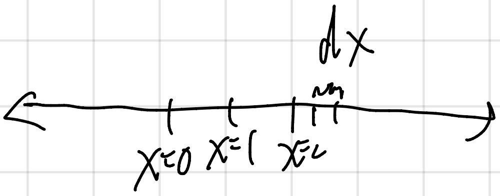
Section 8.1   Arc length

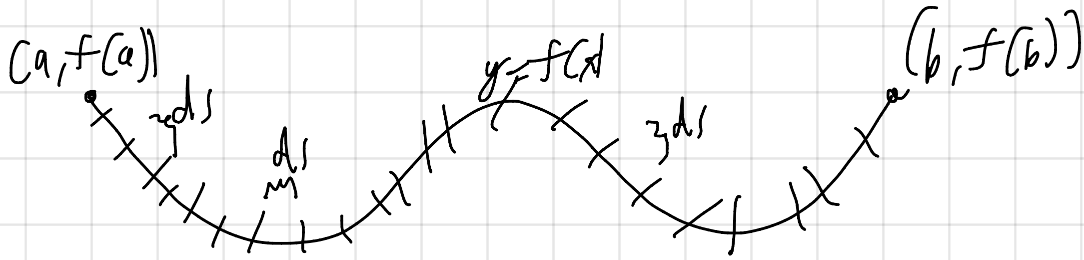
what is the length of the curve $y = x^2$ from $(0, 0)$ to $(1, 1)$.



What is the circumference of a circle of radius R?

What is the length of the curve $y = f(x)$ from $(a, f(a))$ to $(b, f(b))$?





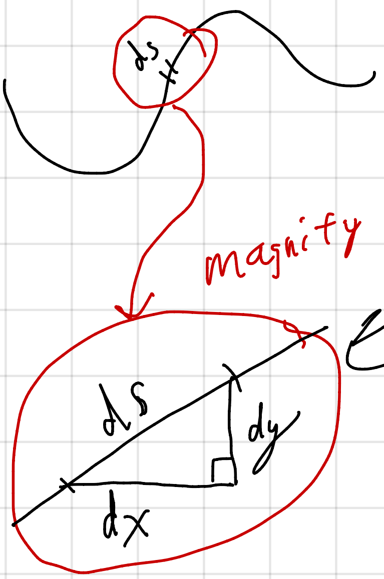
We take our curve and we divide it into many many tiny pieces. We let ds be the length of each tiny piec

The length of the curve is the sum of the lengths of the pieces:

$$\int_{x=a}^{x=b} ds$$

To get an integral we can actually evaluate, we need to rewrite

$ds =$ blah blah blah dx



$$\boxed{(ds)^2 = (dx)^2 + (dy)^2}$$

$$ds = \sqrt{(dx)^2 + (dy)^2}$$

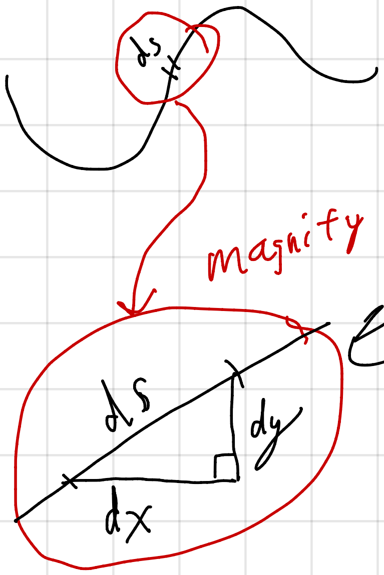
$$ds = \sqrt{(1 + (dy/dx)^2)(dx)^2}$$

The length of the curve is the sum of the lengths of the pieces:

$$\int_{x=a}^{x=b} ds$$

To get an integral we can actually evaluate, we need to rewrite

$ds = \text{blah blah blah } dx$



$$\boxed{(ds)^2 = (dx)^2 + (dy)^2}$$

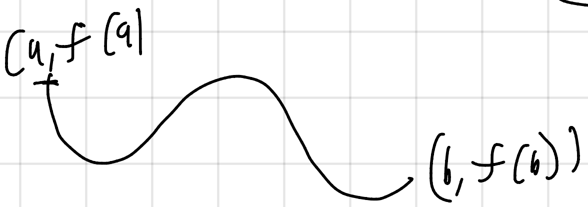
$$ds = \sqrt{(dx)^2 + (dy)^2}$$

$$ds = \sqrt{\left(1 + (dy/dx)^2\right)(dx)^2}$$

$$\boxed{ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx}$$

If we integrate from left to right, $dx > 0$

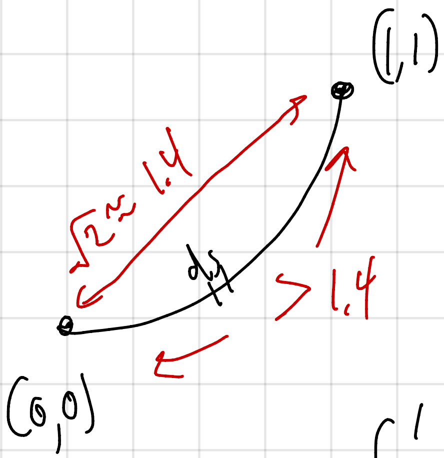
$\sqrt{(dx)^2} = dx$



The length of the curve $y = f(x)$ from $x = a$ to $x = b$ is

$$\int_a^b ds = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

Find the length of $y=x^2$ from $(0,0)$ to $(1,1)$



$\frac{dy}{dx}=2x$

$$\int_{x=0}^{x=1} ds = \int_0^1 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$= \int_0^1 \sqrt{1+(2x)^2}\,dx$$

$2x=\tan\theta$

$\sqrt{1+(2x)^2}=\sec\theta$

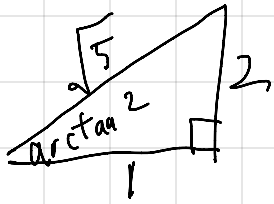
$x=\frac{1}{2}\tan\theta$

$dx=\frac{1}{2}\sec^2\theta\,d\theta$

$$= \int_0^{\arctan 2} \sec\theta \quad \frac{1}{2}\sec^2\theta\,d\theta$$

$\|$ skip to answer

$\Downarrow$



$\tan\theta = 2x$

$\theta = \arctan 2x$

$$\frac{1}{2}\cdot\frac{1}{2}\left[\sec\theta\tan\theta + \ln|\sec\theta+\tan\theta|\right]_0^{\arctan 2}$$

$$\frac{1}{4}\left[\sqrt{5}\cdot 2 + \ln|\sqrt{5}+2|\right] - \cancel{\sec 0\tan 0} - \ln|\cancel{1}+0|$$

$\approx 1,47$

ds = the length of a tiny piece of a curve

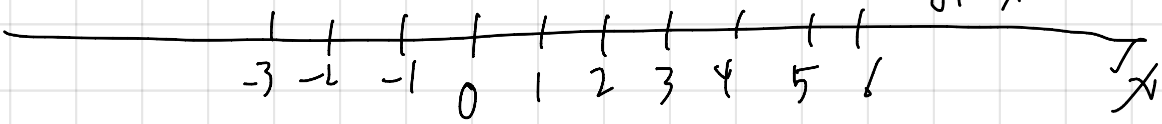
$$(ds)^2 = (dx)^2 + (dy)^2$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

If ds represents the length of a tiny piece of a curve, what does s represent?

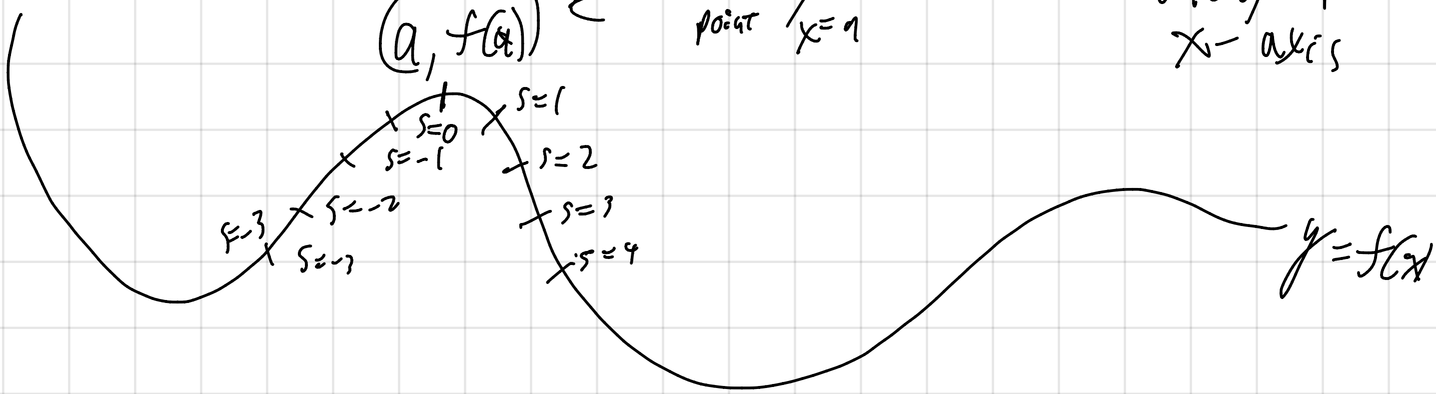
dx = The length of a tiny piece of x-axis

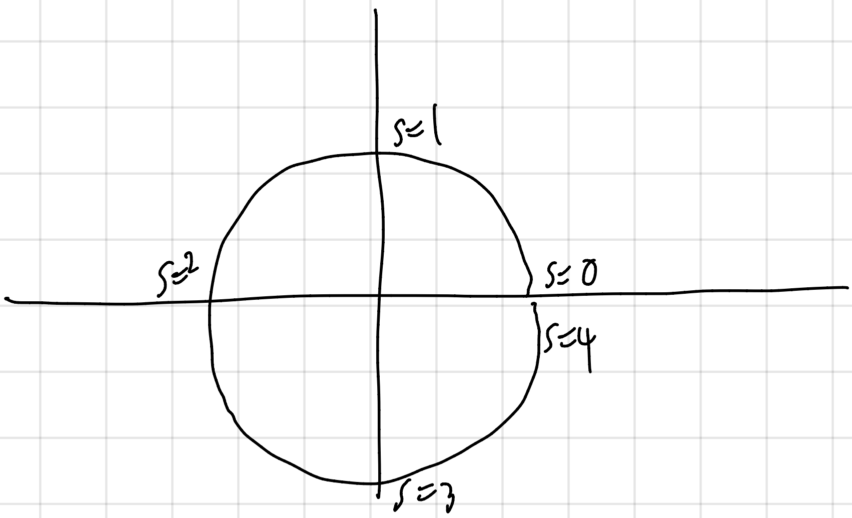
The number line (x-axis)



x represents position along the x-axis

$(a, f(a)) \leftarrow$ arbitrary point / $x = a$



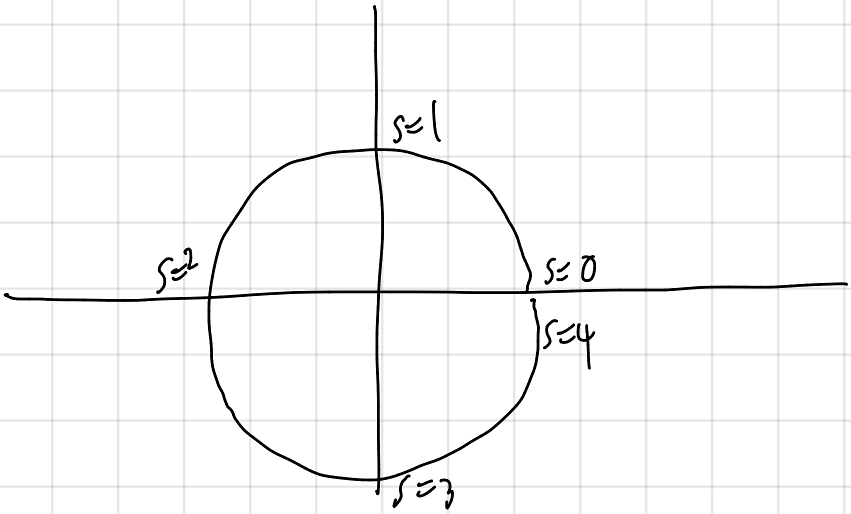


Circle, radius $= \frac{2}{\pi}$

circumference

$= 2\pi \cdot R =$

$2\pi \cdot \frac{2}{\pi} = 4$

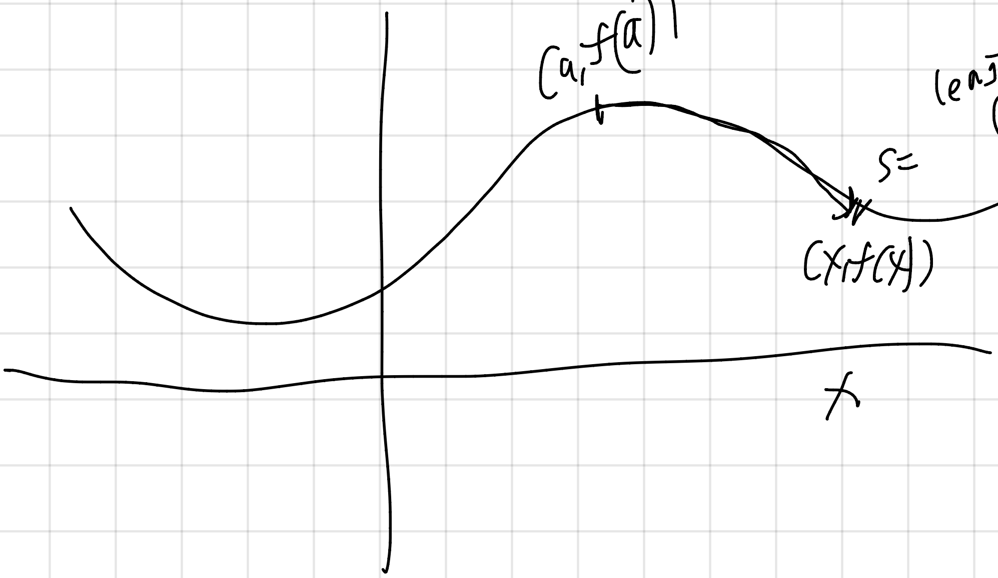


Circle, radius $= \frac{2}{\pi}$

circumference

$= 2\pi \cdot R =$

$2\pi \cdot \frac{2}{\pi} = 4$

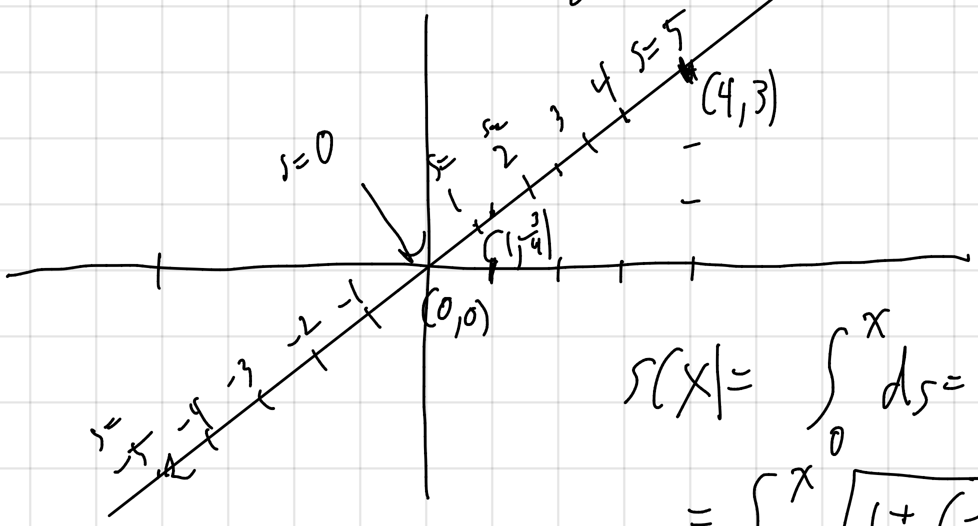


s = length of curve from (a, f(a)) to (x, f(x))

Example

$y = \frac{3}{4}x = f(x)$

$f'(x) = \frac{3}{4}$



$$S(x) = \int_0^x ds = \int_0^x \sqrt{1 + f'(x)^2}\, dx$$

$$= \int_0^x \sqrt{1 + \left(\frac{3}{4}\right)^2}\, dx = \int_0^x \frac{5}{4}\, dx$$

$$S(x) = \frac{5}{4}x$$

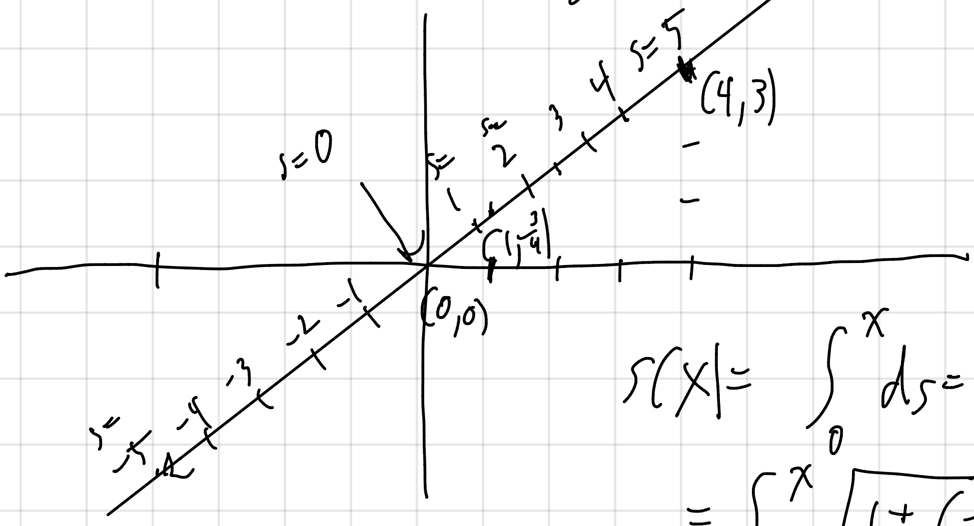
$x = -4$

$S(-4) = -5$

Example

$y = \frac{3}{4}x = f(x)$

$f'(x) = \frac{3}{4}$



$$s(x) = \int_0^x ds = \int_0^x \sqrt{1+f'(x)^2}\,dx$$

$$= \int_0^x \sqrt{1+\left(\frac{3}{4}\right)^2}\,dx = \int_0^x \frac{5}{4}\,dx$$

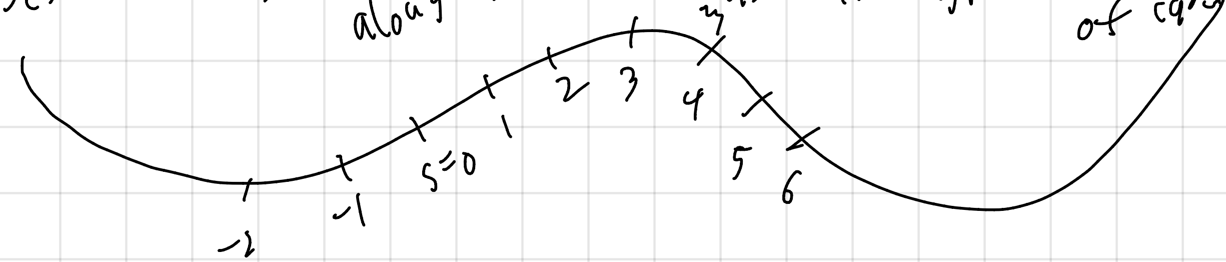
$s(\overset{x=-4}{-4}) = -5$

$$s(x) = \frac{5}{4}x$$

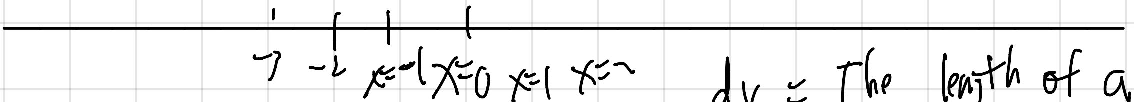
$y = f(x)$

s = length function along a curve

ds = the length of a tiny bit of curve



x represents length function along the line



dx = The length of a tiny bit of line