Math 1B (8:30AM) 2 March 2020

8.1 #9

9–20 Find the exact length of the curve.

9. $y = 1 + 6x^{3/2}, \quad 0 \leqslant x \leqslant 1$

$$\int_0^1 ds = \int_0^1 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = 6 \cdot \frac{3}{2} x^{\frac{1}{2}} = 9x^{\frac{1}{2}}$$

$$\left(\frac{dy}{dx}\right)^2 = 81x$$

$$= \int_0^1 \sqrt{1+81x}\, dx$$

$$= \left[\frac{1}{81}\cdot\frac{2}{3}(1+81x)^{3/2}\right]_0^1$$

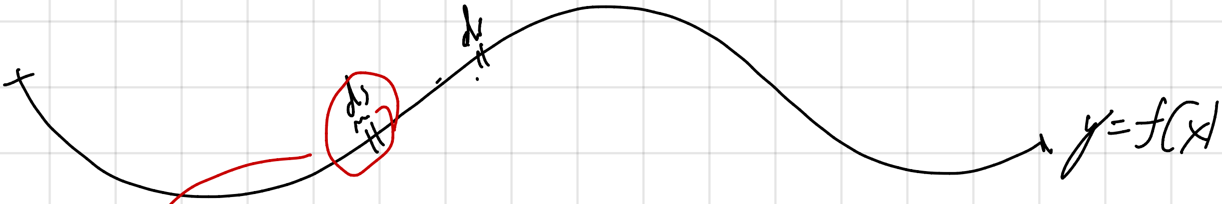
$$\int f(x)\,dx = F(x)$$

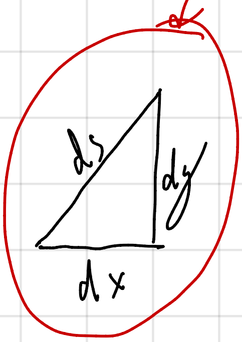
$$\int f(mx+k)\,dx = \frac{1}{m}F(mx+k)$$

$$\int \sqrt{x}\,dx = \frac{2}{3}x^{\frac{3}{2}}$$

$$\frac{2}{243}\left(82^{3/2} - 1\right)$$

$$\frac{2}{243}\left[(1+81\cdot 1)^{3/2} - (1+81\cdot 0)^{3/2}\right]$$





ds = length of a tiny piece of the curve $y=f(x)$

$$ds = \sqrt{(dx)^2 + (dy)^2} = \sqrt{1 + (dy/dx)^2}\, dx$$

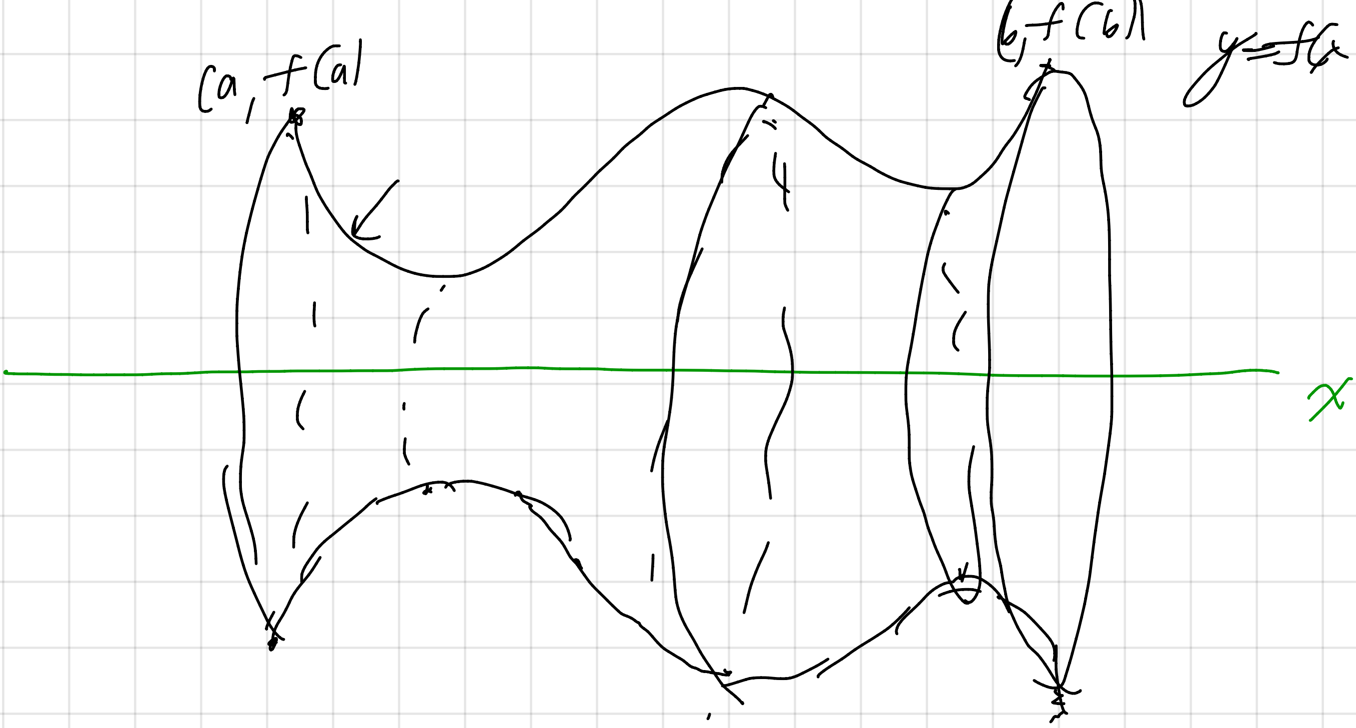
$$ds = \sqrt{(dx/dy)^2 + 1}\, dy$$

$$x = g(y)$$

$$\frac{dx}{dy} = g'(y)$$

Section 8.2

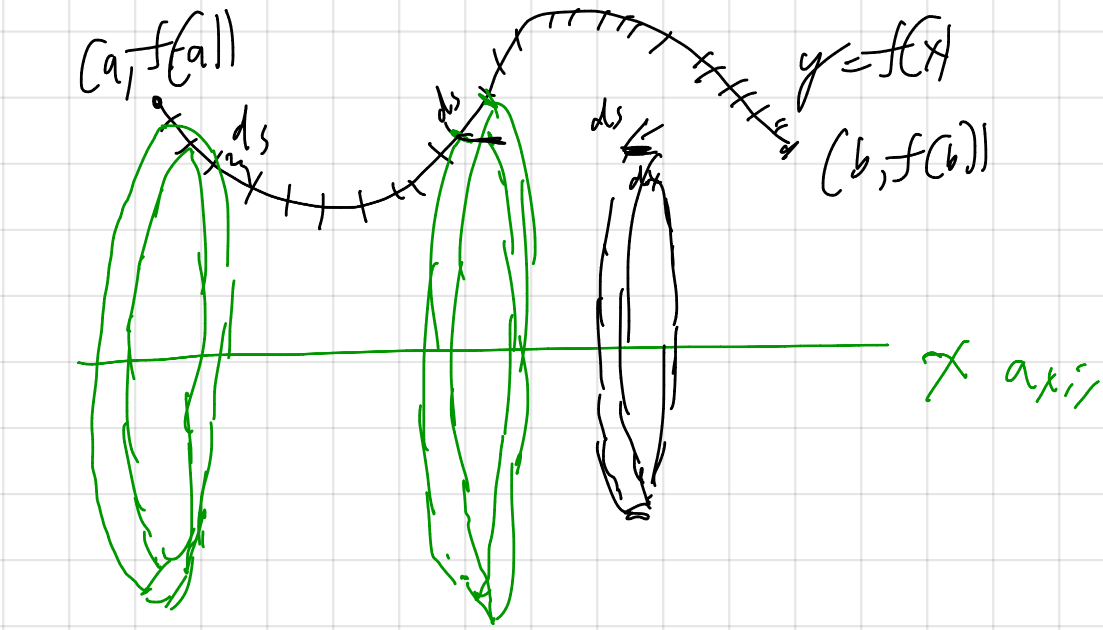
Suppose we take the graph of a positive function $y=f(x)$ from $(a, f(a))$ to $(b, f(b))$ $(a<b)$



and rotate it about the x-axis

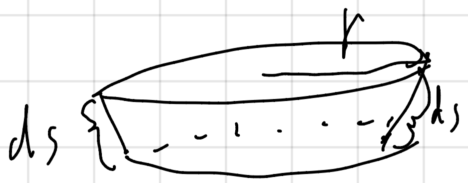
The result will be a surface, a "surface of revolution"

How can we find the area of the resulting surface revolution,

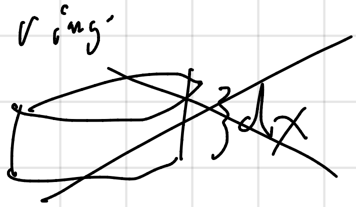


We divide our curve into many tiny pieces, each of length ds. We then rotate each tiny piece about the x axis to get a "slanted ring"

We find the area of the surface of revolution by adding the areas of the rings (using an integral)



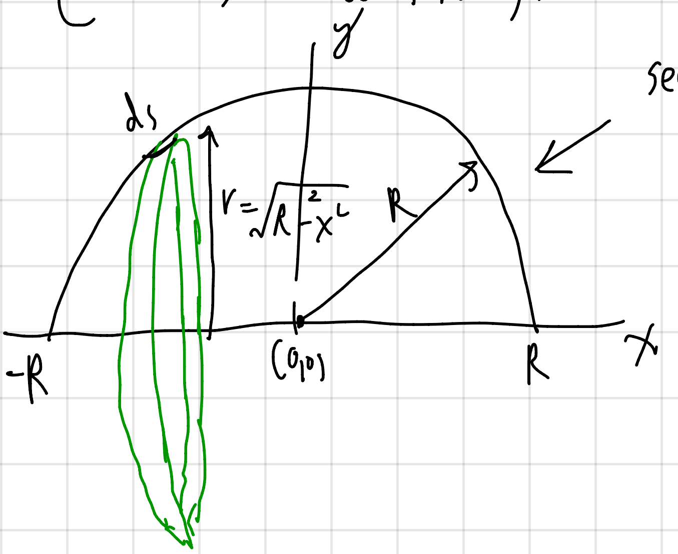
Formula for the area of a slanted ring:



= Circumference · ds

$= \boxed{2\pi r\, ds}$

Find the surface area of a sphere of radius R (R is a constant).



semi-circle of Radius R

$$y = \sqrt{R^2 - x^2}$$

(Rotate this semi-circle about the x-axis to get a surface of revolution, a sphere of radius R)

$$2\pi \int_{x=-R}^{x=R} r \, ds = 2\pi \int_{-R}^{R} \sqrt{R^2-x^2} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \, dx$$

$$= 2\pi \int_{-R}^{R} \sqrt{R^2-x^2} \sqrt{1 + \frac{x^2}{R^2-x^2}} \, dx$$

$$y = \sqrt{R^2-x^2}$$

$$\frac{dy}{dx} = \frac{-\cancel{2}x}{\cancel{2}\sqrt{R^2-x^2}}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{x^2}{R^2-x^2}$$

$$\sqrt{A}\,\sqrt{B} = \sqrt{AB}$$

$$= 2\pi \int_{-R}^{R} \sqrt{R^2 - x^2 + x^2} \, dx = 2\pi \int_{-R}^{R} \sqrt{R^2} \, dx$$

$$= 2\pi \int_{-R}^{R} R \, dx = 2\pi R \cdot (2R)$$

$R-(-R)$

$$= \boxed{4\pi R^2}$$

$$\int_a^b K = K(b-a)$$

8.1

20. $y = 1 - e^{-x}, \quad 0 \leqslant x \leqslant 2$

Find the length of $y = 1 - e^{-x}$ from $(0,0)$ to $(2, 1-e^{-2})$.

$$\int_{x=0}^{x=2} ds = \int_0^2 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$y = 1 - e^{-x}$$
$$\frac{dy}{dx} = e^{-x}$$
$$\left(\frac{dy}{dx}\right)^2 = e^{-2x}$$

$$= \int_0^2 \sqrt{1 + e^{-2x}}\, dx$$

$$= \int_{\sqrt{2}}^{\sqrt{1+e^{-4}}} u \cdot \frac{-u}{u^2-1}\, du$$

$$u = \sqrt{1+e^{-2x}}$$

x	u
0	$\sqrt{2}$
2	$\sqrt{1+e^{-4}}$

$$u^2 = 1 + e^{-2x}$$
$$u^2 - 1 = e^{-2x}$$
$$\ln(u^2-1) = -2x$$
$$x = -\frac{1}{2}\ln(u^2-1)$$
$$dx = -\frac{1}{2}\frac{\cancel{2}u\, du}{u^2-1} = \frac{-u}{u^2-1}\, du$$

$$= \int_{\sqrt{1+e^{-4}}}^{\sqrt{2}} \frac{u^2}{u^2-1}\, du$$

$$= \int_{\sqrt{1+e^{-4}}}^{\sqrt{2}} 1 + \int_{\sqrt{1-e^{-4}}}^{\sqrt{2}} \frac{1}{u^2-1}\, du$$

$$= \int_{\sqrt{1+e^{-4}}}^{\sqrt{2}} 1\, du + \frac{1}{2}\int_{\sqrt{1+e^{-4}}}^{\sqrt{2}} \frac{1}{u+1}\, du - \frac{1}{2}\int_{\sqrt{1+e^{-4}}}^{\sqrt{2}} \frac{1}{u-1}\, du$$