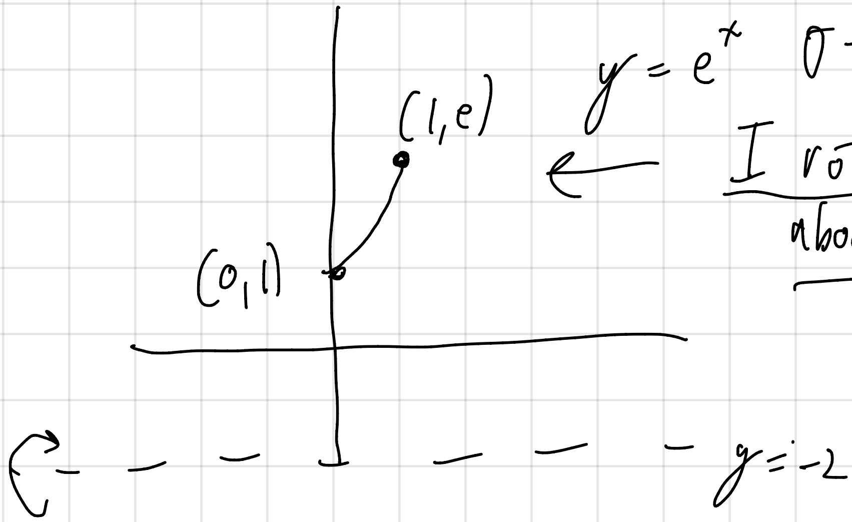
Math (8:30AM) 3 March 2020

We can rotate a curve about the x or y axis (or about a horizontal or vertical axis). We can integrate wrt x or wrt to y.

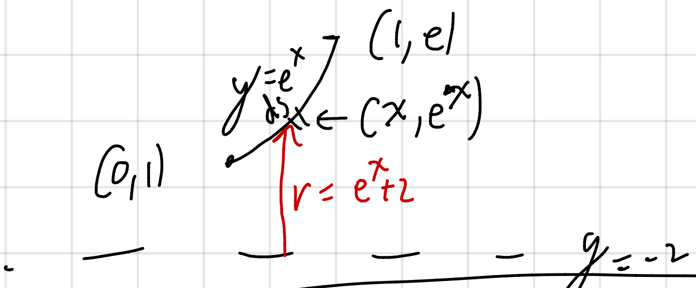


$y = e^x \quad 0 \le x \le 1$

I rotate this curve about the line $y=-2$

Write an integral representing the area of the resulting surface of revolution.

integral wrt x



$$2\pi \int_{x=0}^{x=1} r\, ds = 2\pi \int_0^1 (e^x+2)\sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

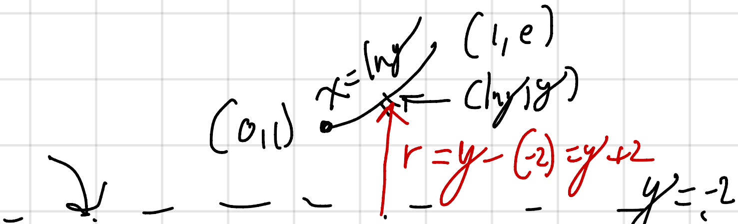
$y=e^x \quad \frac{dy}{dx} = e^x \quad \left(\frac{dy}{dx}\right)^2 = e^{2x}$

$$= \boxed{2\pi \int_0^1 (e^x+2)\sqrt{1+e^{2x}}\, dx}$$

$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

$ds = \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy \qquad ds = \sqrt{(dx)^2+(dy)^2}$

integral wrt y



$$2\pi \int_{y=1}^{y=e} r\, ds$$

$$2\pi \int_1^e (y+2)\sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \boxed{2\pi \int_1^e (y+2)\sqrt{1+\frac{1}{y^2}}\, dy}$$

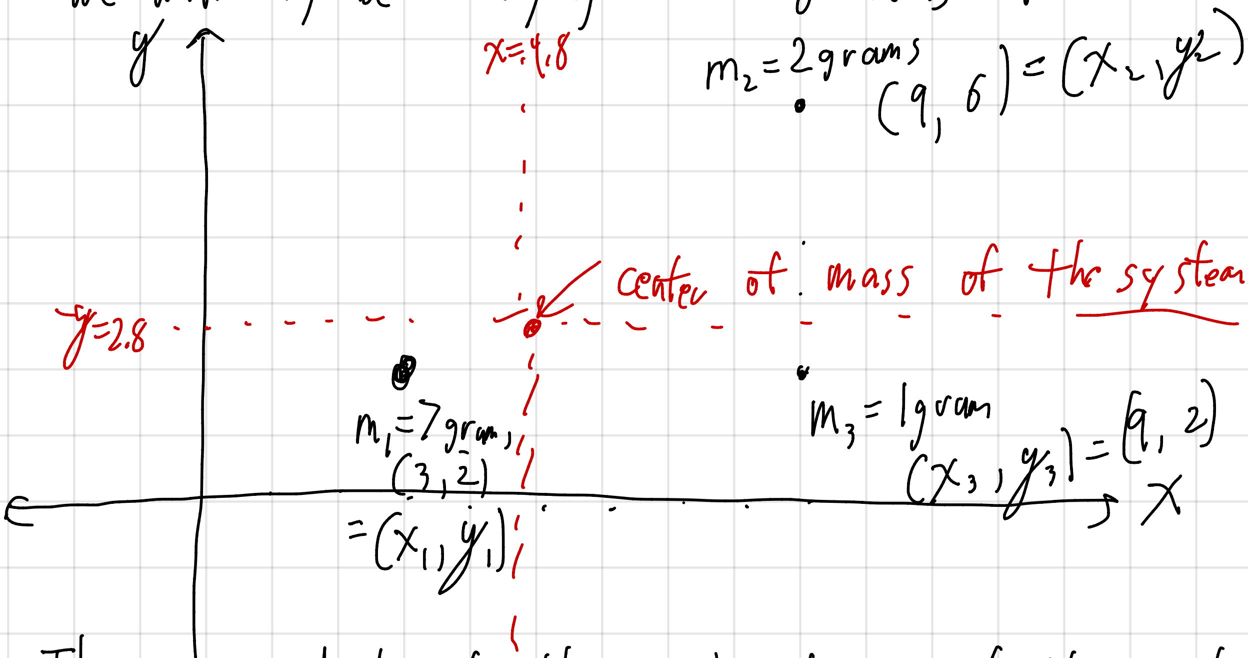
$x = \ln y$

$\frac{dx}{dy} = \frac{1}{y}$

$\left(\frac{dx}{dy}\right)^2 = \frac{1}{y^2}$

Section 8.3

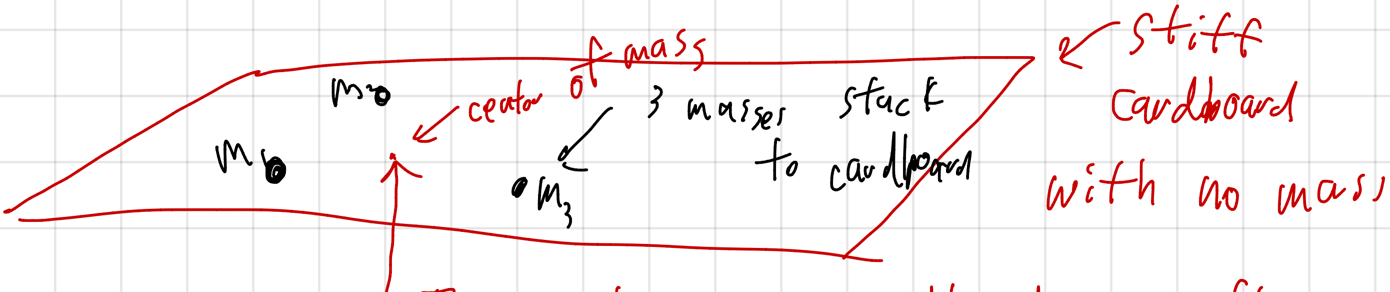
We will only be studying center of mass from 8.3



The x-coordinate of the center of mass of this system, $\bar{x}$ is defined to be the weighted average of the x-coordinates of the masses

$$\bar{x}=\frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3}=\frac{7\cdot 3+2\cdot 9+1\cdot 9}{7+2+1}=4.8$$

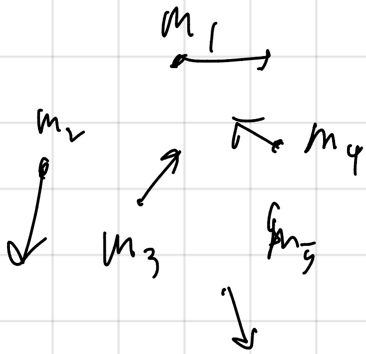
$$\bar{y}=\frac{m_1y_1+m_2y_2+m_3y_3}{m_1+m_2+m_3}=\frac{7\cdot 2+2\cdot 6+1\cdot 2}{10}=2.8$$



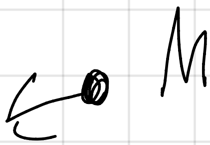
I can balance the cardboard on my finger if I support it under this point Center of mass

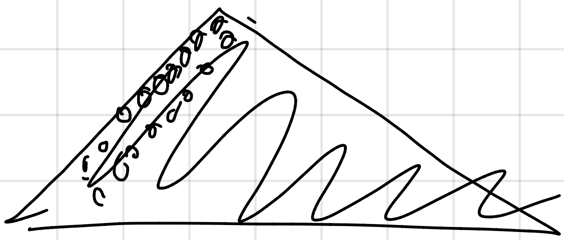
~~#~~ In physics we

In physics we can take a complex system of many masses



and treat it has a single mass $M = m_1 + m_2 + m_3 + m_4 + m_5$ whose position is the center of mass of the system

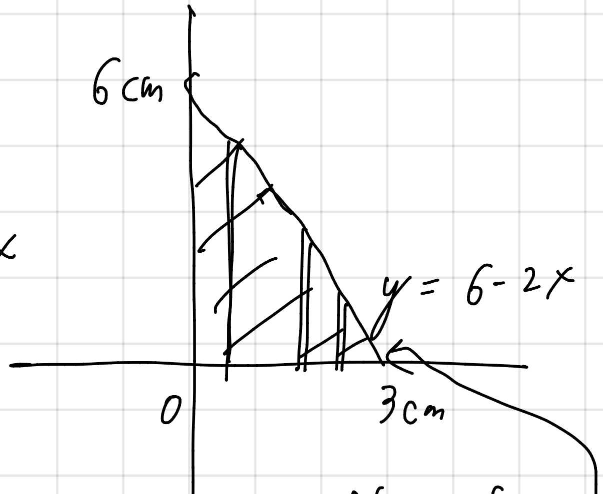




what is the center of mass of this triangle?

$0 \le x \le 3$

$0 \le y \le 6-2x$



what is the center of mass of this shape

assuming it has a uniform density of $6 \frac{gm}{cm^2}$