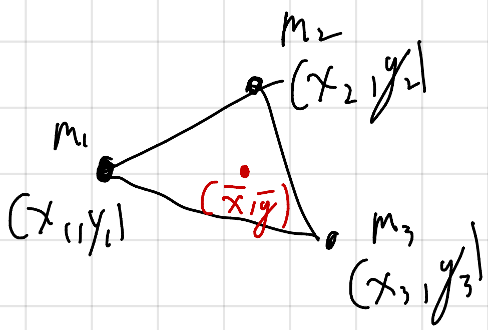
Math 1B (8:30 AM)

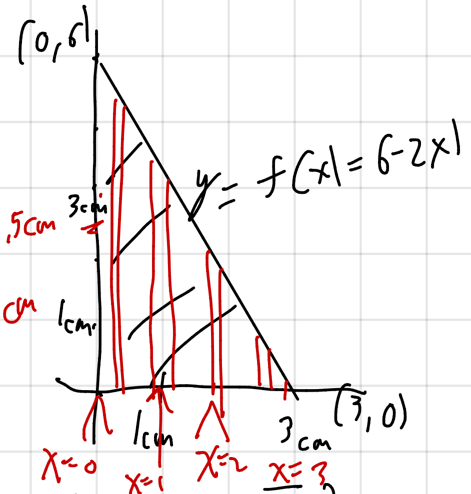
4 March 2020



$$\bar{x} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$\bar{y} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}, \quad \bar{y} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$



$0\,\mathrm{cm} < \bar{x} < 1.5\,\mathrm{cm}$

$0 < \bar{y} < 3\,\mathrm{cm}$

$0 \le x \le 3$

$0 \le y \le 6-2x$

Suppose the density of this region is given by

$\sigma \ \mathrm{gm/cm^2}$

How do I find $\bar{x}$?

Divide shape into vertical rectangles

$$\bar{x} = \frac{\int_0^3 x\, \sigma (6-2x)\,dx}{\int_0^3 \sigma (6-2x)\,dy}$$

mass of each rectangle

density · Area

If $\sigma = 2$ that means a single square cm, (cm^2) $\square$ of material of triangle has mass 2 grams.

$$\bar{x} = \frac{\sigma \int_0^3 x(6-2x)\,dx}{\sigma \int_0^3 (6-2x)\,dx} = \frac{\int_0^3 x(6-2x)\,dx}{\int_0^3 (6-2x)\,dx} = \frac{\int_0^3 (6x-2x^2)\,dx}{\int_0^3 (6-2x)\,dx}$$

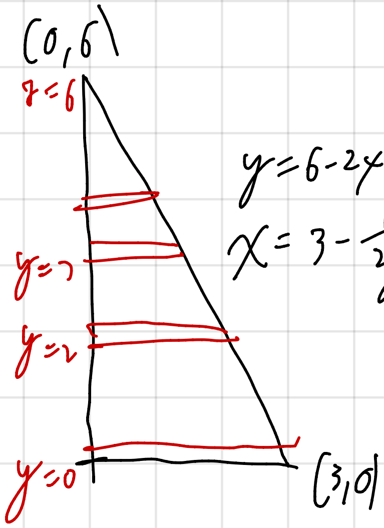
$$= \frac{\left[3x^2 - \frac{2}{3}x^3\right]_0^3}{\left[6x - x^2\right]_0^3} = \frac{27-18}{18-9} = 1 \qquad \bar{x} = 1$$

(2d)

The density of a region $\sigma = \dfrac{\text{mass}}{\text{Area}}$

$\bar{x} = 1$

$\bar{y} = ?$



$y = 6 - 2x$

$x = 3 - \frac{1}{2}y$

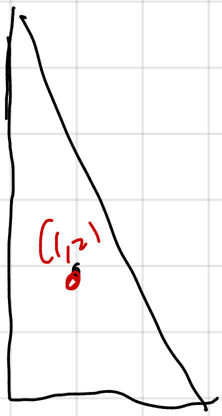
Mass

Area

$$\bar{y} = \frac{\int_0^6 y \,\cancel{\rho}\, \left(3 - \frac{1}{2}y\right) dy}{\int_0^6 \cancel{\rho}\, \left(3 - \frac{1}{2}y\right) dy}$$

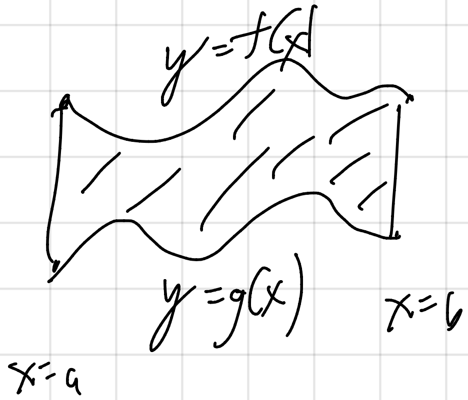
$$= \frac{\int_0^6 \left(3y - \frac{1}{2}y^2\right) dy}{\int_0^6 \left(3 - \frac{1}{2}y\right) dy} = \frac{\left[\frac{3}{2}y^2 - \frac{1}{6}y^3\right]_0^6}{\left[3y - \frac{1}{4}y^2\right]_0^6}$$

$$= \frac{54 - 36}{18 - 9} = \frac{18}{9} = 2$$



$a < b$

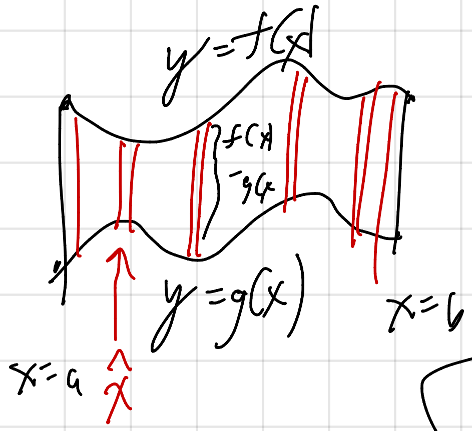
$g(x) \le f(x)$ for $a \le x \le b$



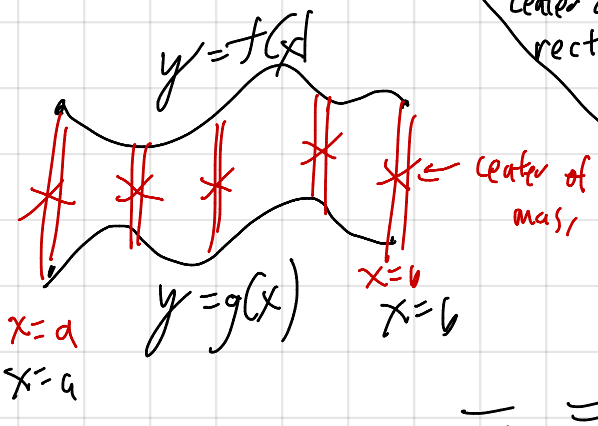
Find the center of mass of the region

$a \le x \le b$

$g(x) \le y \le f(x)$



$$\bar{x} = \frac{\int_a^b x \overbrace{\underbrace{\left(f(x) - g(x)\right)}_{\text{height}} dx}^{\text{area}}}{\int_a^b \left(f(x) - g(x)\right) dx}$$



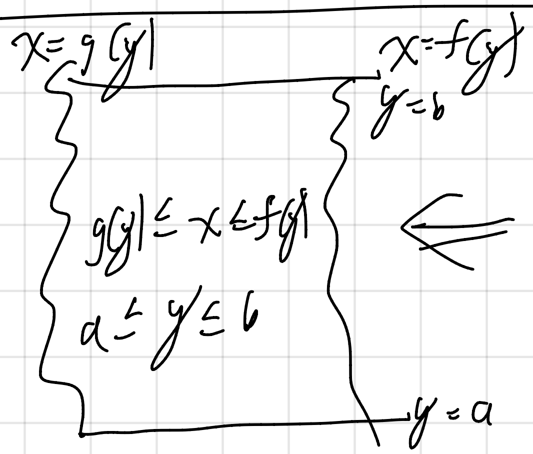
$$\bar{y} = \frac{\int_a^b \overbrace{\frac{f(x) + g(x)}{2}}^{\text{center of mass of each rectangle / y coordinate of } \cancel{\text{rectangle}}} \overbrace{\left(f(x) - g(x)\right) dx}^{\text{area of each rectangle}}}{\int_a^b \left(f(x) - g(x)\right) dx}$$

$$\bar{y} = \frac{\frac{1}{2}\int_a^b \left(\left(f(x)\right)^2 - \left(g(x)\right)^2\right) dx}{\int_a^b \left(f(x) - g(x)\right) dx}$$

Take home exercise

$g(y) \le f(y)$

$a < b$



Find $(\bar{x}, \bar{y})$