Math 1b (8:30AM) 5 Mar 2020

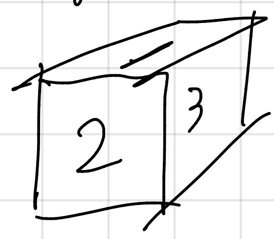
Section 8.5: Probability

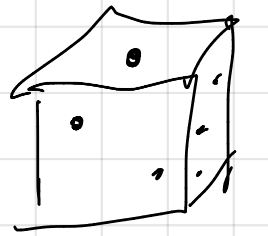
In probability, we assume we conduct an "experiment" with multiple possible outcomes.

Possible experiments

discrete (outcomes can be put in a list)

- I flip a coin:
  possible outcomes (sample space)
  $\{\text{Heads}, \text{Tails}\}$
- I roll a (6-sided) die (singular for dice) and note the outcome
  $\{1, 2, 3, 4, 5, 6\}$

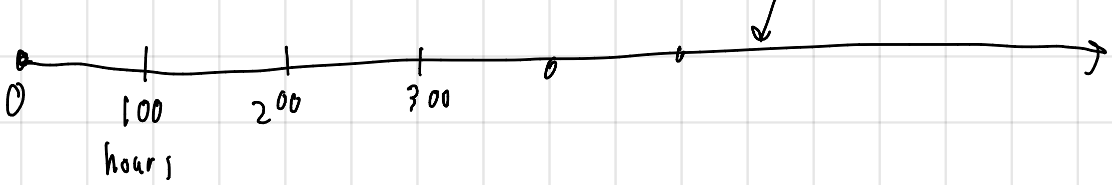




- I roll two dice and note the comes
  $\{11, 12, 13, 14, 15, 16$
  $21, 22, 23, 24, 25, 26$
  $31, 32, 33, \ldots$
  $\vdots$
  $61, 62, 63, 64, 65, 66\}$

Continuous experiment (out comes cannot be listed)

- I turn on a light bulb and I measure how long it takes for it to burn out.
  Possible outcomes: (any non-negative real number)
  562.437114962478 9



A "random variable", usually written as a capital letter, $X, Y, Z, W, D, E, \ldots$

assigns a numerical value to every possible outcome of an experiment.

Technically it is a function from the set of all possible outcomes of an experiment to the real numbers.

Experiment: I flip a coin

Outcomes $\{\text{Heads}, \text{Tails}\}$

$$\text{Heads} \to 0, \quad \text{Heads} \to 17, \quad \text{Tails} \to 1, \quad \text{Tails} \to 3$$

Define $X =$
0 if I get heads
1 if I get tails

Define $Y =$ 17 if I get heads,
3 if I get tails

Experiment: I roll 2 dice

11, 12, 13, 14, 15, 16
21, 22, 23, 24, 25, 26
31, 32, 33, 34, 35, 36
41, 42, 43, 44, 45, 46
51, 52, 53, 54, 55, 56
61, 62, 63, 64, 65, 66

Experiment: I roll 2 dice

						D_1
11,	12,	13,	14,	15,	16	→ 1
21,	22,	23,	24,	25,	26	→ 2
31,	32,	33,	34,	35,	36	→ 3
41,	42,	43,	44,	45,	46	→ 4
51,	52,	53,	54,	55,	56	→ 5
61,	62,	63,	64,	65,	66	→ 6
D_2 ↓ 1	↓ 2	↓ 3	↓ 4	↓ 5	↓ 6	

D_1 = # rolled on first die $\quad$ D_2 = # rolled on 2nd die

$$S = D_1 + D_2$$

45 → $D_1 = 4$

45 → $D_2 = 5$

45 → $S = D_1 + D_2 = 9$

Experiment: I roll one (6-sided) die

D = number rolled,

The probability of an event in an experiment is a number between 0 and 1 (between 0% and 100%) which is the proportion of the time we expect that event to occur if we repeat the experiment many times.

Experiment: I roll one (6-sided) die

D = number rolled,

The probability of an event in an experiment is a number between 0 and 1 (between 0% and 100%) which is the proportion of the time we expect that event to occur if we repeat the experiment many times,

In this example, assuming the die is fair, the probability I roll a 5 is 1/6.

We write this:

$$\underbrace{P}_{\text{The probability}}(\underbrace{D}_{\text{the number rolled on die}} \underbrace{=}_{\text{is}} \underbrace{5}_{5}) \underbrace{=}_{\text{is}} \underbrace{\frac{1}{6}}_{\frac{1}{6}}$$

if $D > 3$ that means, $D=4$ or $D=5$ or $D=6$

$$P(D>3) = P(D=4 \text{ or } D=5 \text{ or } D=6) = \frac{3}{6} = \frac{1}{2} = 50\%$$

Suppose X is a discrete random variable with possible outcomes $x_1, x_2, x_3, \ldots x_n$

Then $P(X=x_1)+P(X=x_2)+P(X=x_3)+\ldots+P(X=x_n)=1$

$$\underset{\text{impossible}}{0} \leq P(X=x_i) \leq \underset{\text{certain}}{1}$$

The "average value" of a random variable X,
or the "expected value"
or the "mean value"

is
$$P(X=x_1)x_1 + P(X=x_2)x_2 + \ldots + P(X=x_n)x_n$$

It is the weighted average of the possible outcomes of the experiments.

Example

D = The number rolled on a fair die

x	$P(D=x)$
1	$1/6$
2	$1/6$
3	$1/6$
4	$1/6$
5	$1/6$
6	$1/6$

Average (or expected or mean) value of D is

$$\frac{1}{6}\cdot 1 + \frac{1}{6}\cdot 2 + \frac{1}{6}\cdot 3 + \frac{1}{6}\cdot 4 + \frac{1}{6}\cdot 5 + \frac{1}{6}\cdot 6$$
$$= 3.5$$

If I roll a die 1000 times, on the average I expect to get a sum of about 3500.

E = # rolled on a weighted die

x	$P(E=x)$
1	1/10
2	1/10
3	1/10
4	1/10
5	1/10
6	1/2

$\frac{1}{2}$

average / expected / mean value of E

$$= \frac{1}{10}\cdot 1 + \frac{1}{10}\cdot 2 + \frac{1}{10}\cdot 3 + \frac{1}{10}\cdot 4 + \frac{1}{10}\cdot 5 + \frac{1}{2}6$$

$$= 4.5$$