Math 1B (8:30AM) 6 March 2020

Experiment
I roll a 6-sided die

(discrete) random variable D = the number rolled on the die

outcomes of X | Probability of each outcome

x	$P(X=x)$
1	$1/6$
2	$1/6$
3	$1/6$
4	$1/6$
5	$1/6$
6	$1/6$

Average (Expected or Mean) value of X

$= P(X=1)\cdot 1 + P(X=2)\cdot 2 + P(X=3)\cdot 3 + \dots P(X=6)\cdot 6$

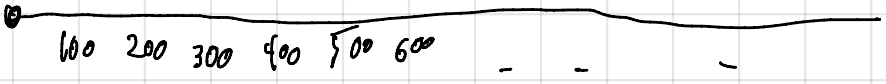
$= \frac{1}{6}\cdot 1 + \frac{1}{6}\cdot 2 + \frac{1}{6}\cdot 3 + \frac{1}{6}\cdot 4 + \frac{1}{6}\cdot 5 + \frac{1}{6}\cdot 6$

$= 3.5$

Experiment: I light a light bulb and time how long it takes to burn out

(continuous) random variable X = time it takes (hours) for bulb to burn out

possible values: all non-negative real numbers



$P(X = 700)$ = The probability our light bulb lasts exactly 700 hours

$X = 700.000000000000\dots000000\dots$ forever

$P(X = 700) = 0$

$P(X = 700)$ = The probability our light bulb lasts exactly 700 hours

$X = 700.000000000000000\ldots$ forever

$P(X = 700) = 0$

By definition, if X is a continuous random variable, then

$$P(X = k) = 0$$

for any particular number k.

There are two ways to model a continuous random variable, X

#1	#2
Use a probability distribution function, f, to model X (Probability Distribution Function = PDF) later	Use a cumulative distribution function (CDF) to model X. We call the CDF for X F and we define $F(x) = P(X \leq x)$
	In our light bulb experiment, the probability a lightbulb lasts no more than 700 hours is $P(X \leq 700) = F(700)$

$$F(x) = P(X \leq x)$$

In our light bulb experiment, the probability a lightbulb lasts no more than 700 hours is

$$P(X \leq 700) = F(700)$$

$$P(X < 700) = P(X \leq 700) - P(X = 700)$$
$$= F(700) - 0 = F(700)$$

$$P(X \leq x) = P(X < x) = F(x)$$

A cumulative distribution function must have the following properties:

$$\lim_{x \to -\infty} F(x) = 0 \qquad \lim_{x \to \infty} F(x) = 1$$

F is increasing, if $s \leq t$ then $F(s) \leq F(t)$

Experiment: I light a light bulb and wait for it to burn out.

X = time it takes for the light bulb to burn out (hours)

F is the CDF of X:

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{1000}, & 0 \leq x \leq 1000 \\ 1, & x > 1000 \end{cases}$$

$F(0) = 0$
$F(500) = 50\% = \frac{1}{2}$
$F(1000) = 100\% = 1$
$F(2000) = 1$

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$F(2000) = 1$

All of my light bulbs last between 0 and 1000 hours with this model

$$F(x) = P(X \le x)$$

$F(1000)$ = The probability a lightbulb lasts less than (or equal to) 1000 hours

The probability a light bulb lasts 400 hours or less is

$$P(X \le 400) = F(400) = \frac{400}{1000} = 40\%$$

$$P(X < 400) = P(X \le 400) - P(X = 400) = 40\% - 0 = 40\%$$

(continuous) $P(X \le x) = P(X < x) = F(x)$

$$P(X > 400) = 1 - P(X \le 400) = 1 - 40\% = 60\%$$

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(continuous) $P(X \leq x) = P(X < x) = F(x)$

$P(X > 400) = 1 - P(X \leq 400) = 1 - 40\% = 60\%$

$P(X \leq K) = F(K)$ $\quad P(X < K) = F(K)$

$P(X \geq K) = 1 - F(K)$ $\quad P(X > K) = 1 - F(K)$

$P(a \leq X \leq b) = P(X \leq b) - P(X < a)$

$P(a \leq X \leq b) = F(b) - F(a)$

The probability a light bulb lasts between 600 and 800 hours is $P(600 \leq X \leq 800) = F(800) - F(600)$

$= \frac{800}{1000} - \frac{600}{1000} = 80\% - 60\% = 20\%$

#1

Use a probability distribution function, f, to model X (Probability Distribution Function = PDF)

The probability distribution function (PDF) of X is a function f,

$$f(x) = F'(x)$$

$$P(a \le X \le b) = \int_a^b f(x)\,dx$$

$$f(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{1000}, & 0 \le x \le 1000 \\ 0, & x > 1000 \end{cases}$$

#2

Use a cumulative distribution function (CDF) to model X. We call the CDF for X F and we define

$$F(x) = P(X \le x)$$

In our light bulb experiment, the probability a lightbulb lasts no more than 700 hours is

$$P(X \le 700) = F(700)$$

$$P(a \le X \le b) = F(b) - F(a)$$

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{1000}, & 0 \le x \le 1000 \\ 1, & x > 1000 \end{cases}$$