Math 16 (8:30 AM) 9 March 2020

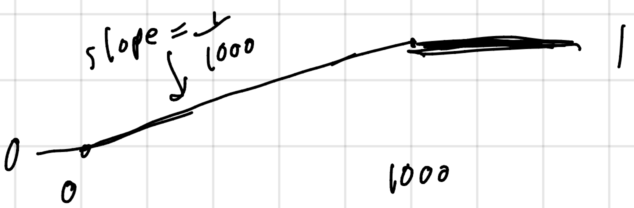
Suppose X is a continuous random variable.

Then there are two ways to represent the distribution of X.

X = lifetime (hours) of a light bulb

① Cumulative Probability Distribution function (CDF).

If F is a CDF for X, then
$$P(X \le b) = F(b)$$



$$P(a \le X \le b) = F(b) - F(a)$$

$0 \le F(x) \le 1$, $F(x)$ is non-decreasing

$\lim_{x \to -\infty} F(x) = 0$ $\lim_{x \to \infty} F(x) = 1$

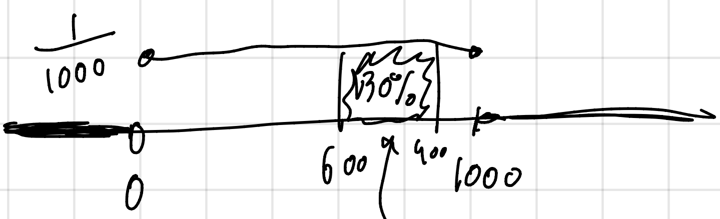
$$F(t) = \begin{cases} 0, & t < 0 \\ \frac{t}{1000}, & 0 \le t \le 1000 \\ 1, & 1000 < t \end{cases}$$

② Probability Distribution function, f

$f = F'$

$$P(a \le X \le b) = \int_a^b f(t)\,dt = F(b) - F(a)$$

$y = f(t)$



What is the probability my light bulb lasts between 600 and 900 hours?

use PDF

$$f(t) = \begin{cases} 0, & t < 0 \\ \frac{1}{1000}, & 0 \le t \le 1000 \\ 0, & 1000 < t \end{cases}$$

$$\int_{600}^{900} f(t)\,dt = \int_{600}^{900} \frac{1}{1000} = \frac{300}{1000} = 30\%$$

Properties of PDF:

$f(t) \ge 0$, $\int_{-\infty}^{\infty} f(t)\,dt = 1$

$f = F'$

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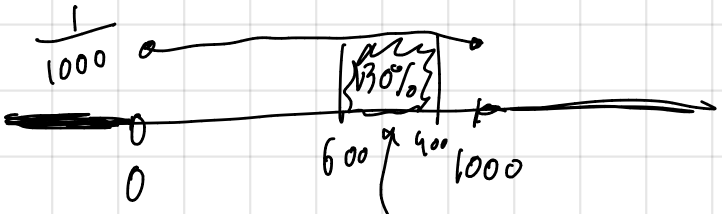

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Properties of PDF:

$f(t) \ge 0$, $\int_{-\infty}^{\infty} f(t)\,dt = 1$

use CDF

$$P(600 \le X \le 900) = F(900) - F(600)$$
$$= \frac{900}{1000} - \frac{600}{1000}$$
$$= \frac{300}{1000} = 30\%$$

$$P(a \le X \le b) = \int_a^b f(x)\,dx$$
$$= F(b) - F(a)$$

$F' = f$

How do I go from the PDF, f, to the CDF, F?

CDF, PDF

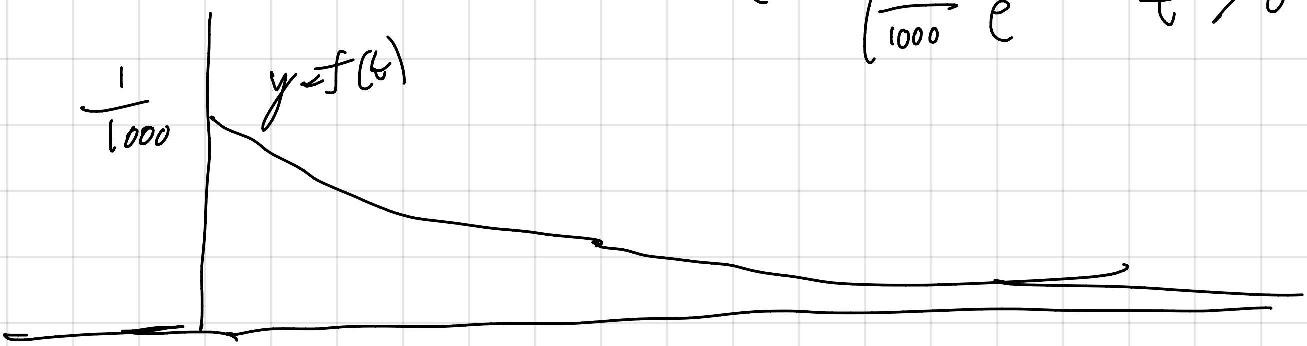
$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt$$

$$\boxed{F(x) = \int_{-\infty}^{x} f(t)\,dt}$$

We light a light bulb until it burns out

X = The time until the light bulb burns out in hours

: PDF of X $= f(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{1000} e^{\frac{-t}{1000}} & t > 0 \end{cases}$



check that $f(t)$ is a well defined PDF:

> $f(t) \geq 0$ for all t ✓

> $\int_{-\infty}^{\infty} f(t)\,dt = 1$

$$\int_{-\infty}^{\infty} f(t)\,dt = \int_0^{\infty} \frac{1}{1000} e^{\frac{-t}{1000}}\,dt = \lim_{t \to \infty} \int_0^t \frac{1}{1000} e^{\frac{-t}{1000}}\,dt$$

$$\lim_{t \to \infty} \left[\frac{-\cancel{1000}}{\cancel{1000}} e^{\frac{-t}{1000}} \right]_0^t = \lim_{t \to \infty} -\overbrace{e^{\frac{-t}{1000}}}^{\to 0} + e^0 = 1$$

Let's find the CDF for X

$$F(t) = P(X \leq t) = \int_{-\infty}^{t} f(s)\,ds$$

If $t \leq 0$ then $F(t) = 0$

if $t > 0$

$$= \int_0^t f(s)\,ds = \int_0^t \frac{1}{1000} e^{\frac{-t}{1000}}\,dt$$

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$$= \int_0^t f(s)\,ds = \int_0^t \frac{1}{1000} e^{\frac{-s}{1000}}\,ds$$

$$= \left[\frac{-1000}{1000} e^{\frac{-s}{1000}}\right]_0^t = -e^{\frac{-t}{1000}} + 1$$

e^0

$$F(t) = 1 - e^{\frac{-t}{1000}}$$

$$F(t) = \begin{cases} 0, & t < 0 \\ 1 - e^{\frac{-t}{1000}}, & t \geq 0 \end{cases}$$

$F(t)$ is non decreasing

$\lim_{t \to \infty} F(t) = 0$

$\lim_{t \to \infty} F(t) = 1$

Average value of a discrete random variable, D with outcomes $x_1, x_2, \ldots x_n$ is

$$P(X = x_1)x_1 + P(X = x_2)x_2 + \ldots + P(X = x_n)x_n$$

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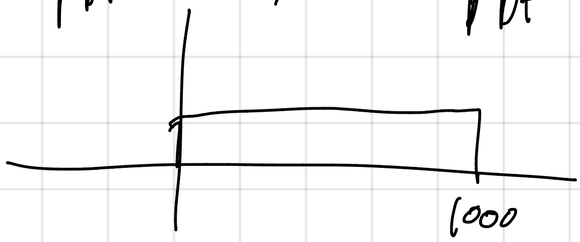
What is the average/mean/expected value of a continuous random variable X, with PDF x?

$$\int_{-\infty}^{\infty} x f(x)\,dx$$

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{1000} & \text{if } 0 \le x \le 1000 \\ 0 & \text{if } x > 1000 \end{cases}$$

PDF for X

PDF



CDF for X

average/expected value of X =

$$\int_{-\infty}^{\infty} x f(x)\,dx = \int_{0}^{1000} x \cdot \frac{1}{1000} = \frac{1}{1000}\frac{1}{2}x^2 \Big|_0^{1000} = \frac{1000^2}{2 \cdot 1000}$$

$$= 500 \text{ hours}$$