Math 1b (8:30AM)

16 March 2020

An example of a differential equation is

$$y' = x+y \qquad \left(y' = \frac{dy}{dx}\right)$$

A solution:

$$\cancel{y = -x-1}$$

Check solution:

$$(-x-1)' = x + (-x-1)$$
$$-1 = -1 \checkmark$$

Yes $y = -x-1$ is a solution to $y' = x+y$

$c=0$

The general solution:

$$\boxed{y = -x-1+ce^x}$$

(c = any constant)

$$y' = x+y$$
$$(-x-1+ce^x)' = x + (-x-1+ce^x)$$
$$-1+ce^x = -1+ce^x \checkmark$$

Many times a differential equation is given together with "initial conditions", extra properties the solution you find must have.

$$y' = x+y \qquad \text{when } x=2,\ y \text{ must equal } 1$$

$$y\big|_{x=2} = 1, \quad y(2) = 1$$

when $x=2$

First find the general solution

$$y = -1-x+ce^x$$
$$1 = -1-2+ce^2$$
$$4 = ce^2 = \quad c = 4e^{-2} = \frac{4}{e^2}$$

$$\boxed{y = -1-x+4e^{-2}e^x}$$

$$\boxed{y = -1-x+4e^{x-2}}$$

$$1 \stackrel{?}{=} -1-2+4e^{2-2}$$
$$1 = -3+4 \checkmark$$

$$y' = x+y$$
$$(-1-x+4e^{x-2})' = x + (-1-x+4e^{x-2})$$
$$-1+4e^{x-2} = -x+4e^{-x-2}$$

Fundamentally there are three ways of thinking about differentiale quations

9.2 ~~slope fields~~ direction fields — (1) Conceptually
Thinking (sometimes visually) about how solutions to differential equations.

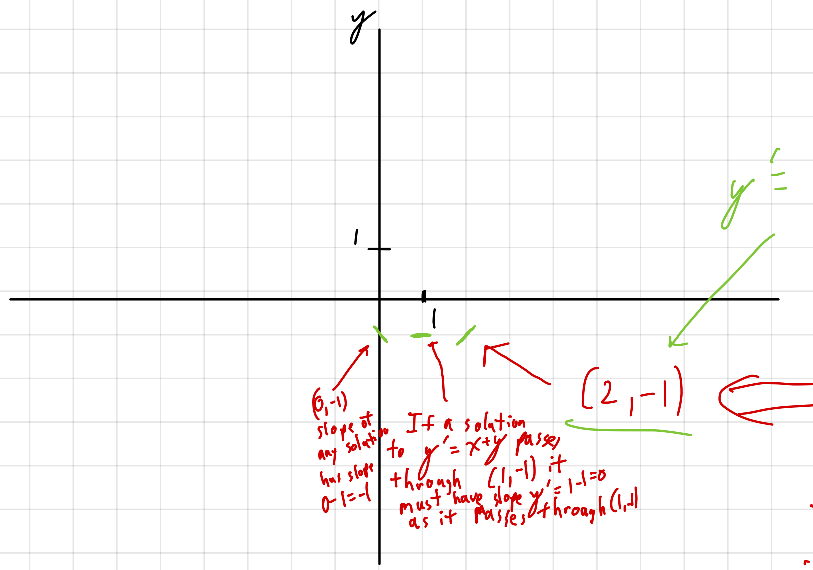
~~9.2~~ Euler's Method — (2) **Numerically**
Using a computer to compute values of the solution to a differential equation to any desired degree of accuracy

9.3 separation of variables — (3) Algebraically - Finding the exact solution to a differential equations

$$y' = 2x \qquad \text{solution: } y = x^2 + c$$

9.2 Direction Fields

$$y' = x + y \qquad y(0) = 2$$



$y' = x + y$

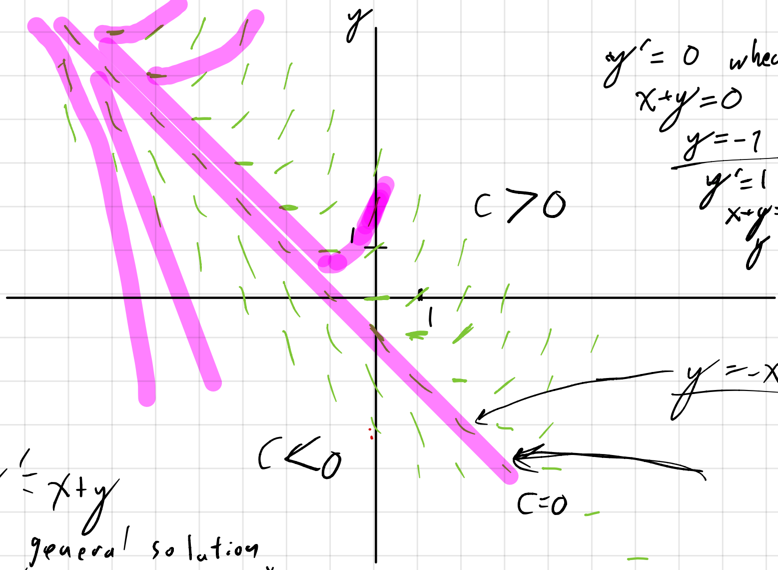
$y'\big|_{(2,-1)} = 2 - 1 =$

If a solution to $\boxed{y' = x+y}$ passes through this point, it must have slope ___ at this point

$y' = x + y$

general solution

$y = -x - 1 + ce^{x}$



$y' = 0$ when

$x + y = 0$

$y = -1$

$y' = 1$ when

$x + y = 1$

$y = 1 - x$

$y = -x - 1$