

Math 16 (8:30AM)

17 March 2020

Section 9.3

Solving differential equations using separation of variables.

$$y' = \frac{2y}{x}$$

Step #1
rewrite $y' = \frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{2y}{x}$$

$$\frac{dy}{y} = \frac{2dx}{x}$$

Step #2

separate the variables, getting all occurrences of x on one side and y on the other.

Step #3
integrate both sides

$$\int \frac{dy}{y} = \int \frac{2}{x} dx$$

$$\ln y = 2 \ln x + C$$

$$e^{\ln y} = e^{2 \ln x + C}$$

$$y = e^C \cdot e^{2 \ln x}$$

$$y = C_2 x^2$$

technically, $\int \frac{dx}{x}$ cannot be integrated but don't worry about it

Step #4

solve for y
(this is not always possible)

$$e^{B \ln A} = A^B$$

we only need one constant

$$\ln y + C_1 = 2 \ln x + C_2$$

$$\ln y = 2 \ln x + (C_2 - C_1)$$

$$\ln y = 2 \ln x + C$$

$$y' = \frac{2}{x} y$$

general solution
check our answer

$$(C x^2)' \stackrel{?}{=} \frac{2}{x} (C x^2)$$

$$2 C x = 2 C x \quad \checkmark$$

$$y' = \frac{2}{x} y$$

$$\frac{dy}{dx} = \frac{2}{x} y$$

$$\frac{dy}{y} = \frac{2}{x} dx$$

$$\int \frac{dy}{y} = \int \frac{2}{x} dx$$

$$\ln|y| = 2 \ln|x| + C_1$$

$$e^{\ln|y|} = e^{2 \ln|x| + C_1}$$

$$|y| = |x|^2 e^{C_1}$$

$$|y| = x^2 C_2 \quad C_2 \geq 0$$

$$y = \pm C_2 x^2$$

$$y = C_3 x^2 \quad C_3 = \pm C_2 \quad C_3 > 0 \text{ or } C_3 < 0 \quad C_3 \neq 0$$

$$y = C_4 x^2 \quad C \text{ any real number}$$

When solving differential equations, you do not need this level of detail.

Let's solve this again, but being very precise (You do not need to do this when solving differential equations)

When we divided both sides by y , we implicitly assumed $y \neq 0$. So we should check if $y=0$ is a solution separately.

$$y' = \frac{2}{x} y$$

$$y=0: (0)' = \frac{2}{x} 0$$

$$0=0 \quad \checkmark$$

$y=0$ is a solution

Problem:

$$y' = \frac{2}{x} y, \quad y(1) = 5$$

$$\frac{dy}{dx} = \frac{2}{x} y$$

$$\int \frac{dy}{y} = \int \frac{2}{x} dx$$

$$\ln y = 2 \ln x + C$$

$$y = C x^2$$

$$5 = C \cdot 1^2$$

$$C = 5$$

$$y(1) = 5$$

$$y = 5x^2$$

Tomorrow in class, make sure you have access to blank sheets of paper, and a phone that can scan and make pdf documents.

$$y' = xy$$

$$\frac{dy}{dx} = xy$$

$$y(1) = 2$$

$$y = e^C e^{\frac{1}{2}x^2}$$

$$y = C e^{\frac{1}{2}x^2}$$

$$2 = C e^{\frac{1}{2}}$$

$$C = \frac{2}{e^{\frac{1}{2}}}$$

$$y = 2e^{-\frac{1}{2}} e^{\frac{1}{2}x^2}$$

$$y = 2e^{\frac{1}{2}(x^2-1)}$$

check

$$(C e^{\frac{1}{2}x^2})' = x(C e^{\frac{1}{2}x^2})$$

$$x C e^{\frac{1}{2}x^2} = x C e^{\frac{1}{2}x^2}$$

$$\int \frac{dy}{y} = \int x dx$$

$$\ln y = \frac{1}{2}x^2 + C$$

$$y = e^{\frac{1}{2}x^2 + C}$$