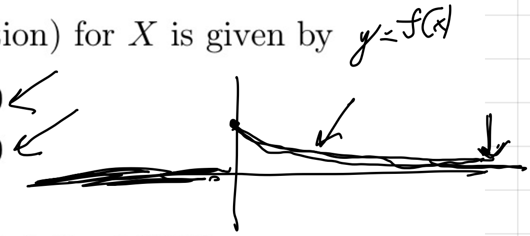
Math 1B (8:30 AM) 20 March 2020

(3) Suppose we wait for a bus, and we let X be the random variable

$$X = \text{The time it takes (in minutes) for the bus to arrive}$$

Suppose that the PDF (probability distribution function) for X is given by

$$f(x) = \begin{cases} 0 & x < 0 \\ ke^{-x/10} & x \geq 0 \end{cases}$$



where k is a constant.

(a) Find the value k must have for $f(x)$ to be a well defined PDF.
(b) Find the probability a bus arrives in 10 minutes or less.
(c) Find the average (mean) amount of time we expect to have to wait for the bus.

(a) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$= \int_0^{\infty} Ke^{-x/10}\,dx = \lim_{t\to\infty} \int_0^t Ke^{-x/10}\,dx = \lim_{t\to\infty} \left[K(-10)e^{-\frac{x}{10}}\right]_0^t$$

$$= \lim_{t\to\infty} -10K\underbrace{e^{-t/10}}_{\to 0} + 10K\underbrace{e^0}_{=1} = 10K = 1$$

$$K = \frac{1}{10}$$

$$f(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{10}e^{-x/10} & x \geq 0 \end{cases}$$

(b) Find the probability a bus arrives in 10 minutes or less.

$$P(X \leq 10) = \int_{-\infty}^{10} f(x)\,dx = \int_0^{10} \frac{1}{10}e^{-x/10}\,dx$$

$$= \left[-\cancel{\frac{10}{10}}\, e^{-\frac{x}{10}}\right]_0^{10} = -e^{-1} + e^0 = 1 - \frac{1}{e}$$

$$\approx 63\%$$

(c) Find the average (mean) amount of time we expect to have to wait for the bus.

$$\int_{-\infty}^{\infty} \underbrace{x f(x)}_{=0 \text{ when } x \leq 0}\,dx = \int_0^{\infty} x \cdot \frac{1}{10}e^{-x/10} = \lim_{t\to\infty} \int_0^t \frac{1}{10}xe^{-\frac{x}{10}}\,dx =$$

	diff	int
$+$	$\frac{1}{10}x$	$e^{-x/10}$
$-$	$\frac{1}{10}$	$-10e^{-x/10}$
$+$	0	$100e^{-x/10}$

$$= \lim_{t \to \infty} \left[-xe^{-x/10} - 10e^{-x/10} \right]_0^t$$

$$= \lim_{t \to \infty} \left[\left(-te^{-t/10} - 10e^{-t/10} \right) - \left(0 - 10e^{0} \right) \right] = 10$$

$$\downarrow 0 \qquad \downarrow 0$$

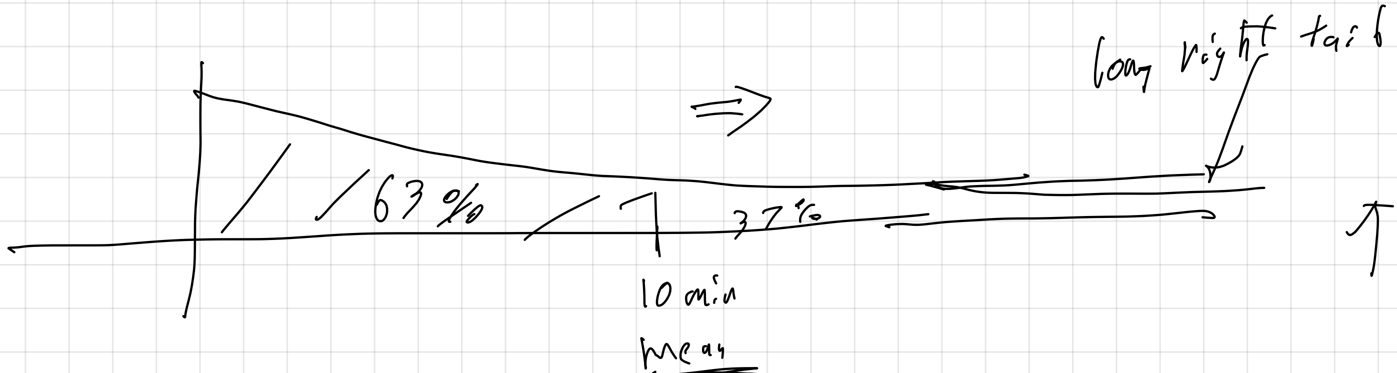
(L'Hospital)

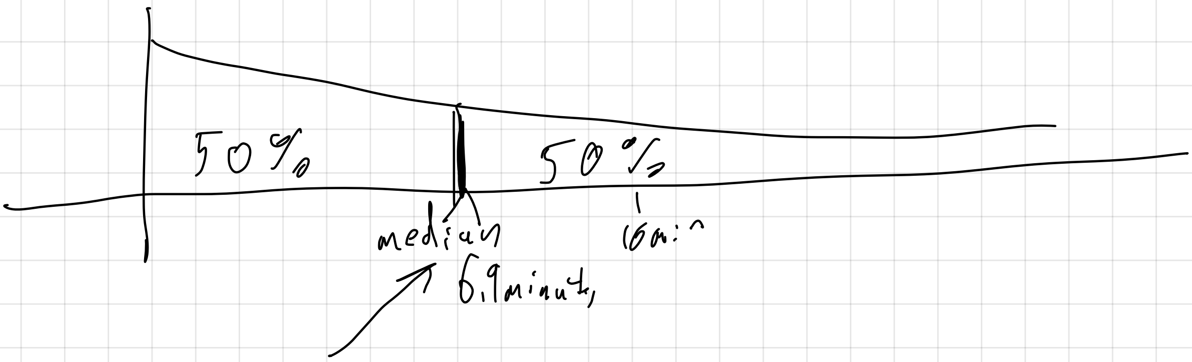
(c) The average time it takes a bus to arrive is 10 minutes.

(b) 63% of the time a bus comes before 10 minutes.

median time it takes for a bus to arrive is m

$P(X \leq m) = 50\%$





What is the median time, m, for the bus to arrive

assume $m \geq 0$

$$P(X \leq m) = 0.5$$

$$\int_{-\infty}^{m} f(x)\,dx = 0.5$$

$$\int_{0}^{m} \frac{1}{10} e^{-x/10}\,dx = 0.5$$

$$\left[-e^{-x/10}\right]_0^m = 0.5$$

$$-e^{-m/10} + 1 = 0.5$$

$$e^{-m/10} = 0.5$$

$$\frac{-m}{10} = \ln 0.5$$

$$m = -10 \ln 0.5 = 10 \ln 2 \approx 6.9$$

minutes