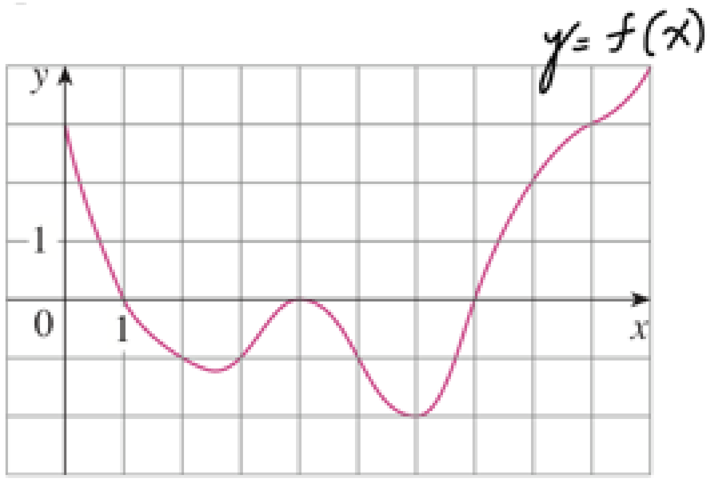
(1) Consider the following function,

$$f(x) = \begin{cases} 6 & -3 \le x < 0 \\ 6 - 2x & 0 \le x < 4 \\ \sqrt{9 - (x-7)^2} & 4 \le x < 10 \\ -\sqrt{1 - (x-11)^2} & 10 \le x \le 12 \end{cases}$$

(a) Sketch a graph of the above function. Graph paper is recomended.
(b) Use your sketch to find the following values
(i) $\int_{-3}^{0} f(x)\ dx$
(ii) $\int_{0}^{-3} f(x)\ dx$
(iii) $\int_{0}^{3} f(x)\ dx$
(iv) $\int_{3}^{4} f(x)\ dx$
(v) $\int_{0}^{4} f(x)\ dx$
(vi) $\int_{0}^{4} |f(x)|\ dx$
(vii) $\int_{4}^{10} f(x)\ dx$
(viii) $\int_{11}^{10} f(x)\ dx$
(ix) $\int_{4}^{11} f(x)\ dx$
(x) $\int_{4}^{11} |f(x)|\ dx$

(2) Calculate $\int_{1}^{-3} |2(x+2)|\ dx$. Show explicitely how you use the laws of integrals to calculate this, splitting up the integral into two pieces appropriately.

(3) Consider the function graphed below:



(a) Estimate the value of $\int_{0}^{10} f(x)\,dx$ using a Riemann sum with 5 sub-intervals taking the sample points to be left endpoints.
(b) Estimate the value of $\int_{0}^{10} f(x)\,dx$ using a Riemann sum with 5 sub-intervals taking the sample points to be right endpoints.
(c) Estimate the value of $\int_{0}^{10} f(x)\,dx$ using a Riemann sum with 5 sub-intervals taking the sample points to be midpoints.
(d) Estimate the value of $\int_{0}^{10} |f(x)|\,dx$ using a Riemann sum with 5 sub-intervals taking the sample points to be left endpoints.
(e) Estimate the value of $\int_{0}^{10} |f(x)|\,dx$ using a Riemann sum with 5 sub-intervals taking the sample points to be right endpoints.
(f) Estimate the value of $\int_{0}^{10} |f(x)|\,dx$ using a Riemann sum with 5 sub-intervals taking the sample points to be midpoints.

(4) Consider $\int_{-2}^{4} x^2 dx$
   (a) Estimate the value of the above integral using a Riemann sum with 3 sub-intervals taking the sample points to be left endpoints.
   (b) Estimate the value of the above integral using a Riemann sum with 3 sub-intervals taking the sample points to be right endpoints.
   (c) Estimate the value of the above integral using a Riemann sum with 3 sub-intervals taking the sample points to be midpoints.
   (d) Write the value of the above integral as a limit of Riemann sums.

(5) What definite integral is equal to $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{1+\left(5+\frac{2k}{n}\right)^2} \cdot \frac{2}{n}$?

(6) Suppose the values of a function are given by the following table

x	6	7	8	9	10	11	12
$f(x)$	12	11	9	7	3	-2	-5

   (a) Write and evaluate a Riemann sum with 3 subdivisions using left endpoints as sample points to estimate $\int_{6}^{12} f(x)\, dx$
   (b) Write and evaluate a Riemann sum with 3 subdivisions using right endpoints as sample points to estimate $\int_{6}^{12} f(x)\, dx$
   (c) Write and evaluate a Riemann sum with 3 subdivisions using midpoints as sample points to estimate $\int_{6}^{12} f(x)\, dx$

(7) Suppose $\int_{2}^{5} f(x)\, dx = 7$ and $\int_{2}^{9} f(x)\, dx = 12$. Find $\int_{5}^{9} f(x)\, dx$. Show explicitely how you use the properties of integrals to find this.

(8) True or false?
   (a) $\int_{0}^{7} xe^x\, dx = x \int_{0}^{7} e^x\, dx$
   (b) $\int_{2}^{5} 3\sin x\, dx = -3 \int_{5}^{2} \sin x\, dx$
   (c) $\int_{a}^{b} (f(x) - 3g(x))\, dx = \int_{a}^{b} f(x)\, dx - 3\int_{a}^{b} g(x)\, dx$
   (d) x is a dummy variable in $\int_{0}^{5} \cos^2(x)\, dx$
   (e) $\int_{2}^{4} \cos(x^2)\, e^x dx = \left(\int_{2}^{4} \cos(x^2)\, dx\right)\left(\int_{2}^{4} e^x\, dx\right)$
   (f) $\int_{0}^{5} \cos(x^2)\, dx = \int_{0}^{5} \cos(t^2)\, dt$