(1) An oven is heating up. Suppose that

$$f(t) = \text{The temperature of the oven } (^\circ F) \text{ after } t \text{ minutes have passed}$$

and

$$g(t) = \text{The rate the temperature of the oven is increasing } (^\circ F/\text{min}) \\ \text{after } t \text{ minutes have passed}$$

Suppose that at time $t = 10$, the temperature of the oven is $90^\circ F$ (in other words, suppose that $f(10) = 90$).

(a) Using $f(t)$ and no other variables, write an expression representing the net increase in temperature of the oven (in $^\circ F$) from $t = 0$ minutes to $t = 30$ minutes.

(b) Using $g(t)$ and no other variables, write an expression representing the net increase in temperature of the oven (in $^\circ F$) from $t = 0$ minutes to $t = 30$ minutes.

(c) Using $g(t)$ and no other variables, write an expression representing the temperature of the oven (in $^\circ F$) at $t = 30$ minutes.

(d) Using $g(t)$ and no other variables, write an expression representing the temperature of the oven (in $^\circ F$) at $t = 0$ minutes.

(2) Consider the definite integral

$$\int_{2}^{26} \sqrt{9+x^2}\, dx$$

(a) Write a Riemann sum approximating the above integral with 4 subdivisions, using left hand endpoints as sample points. You do NOT need to simplify or evaluate the sum, just write it out. You're final sum should not include any variables or letters or a summation sign. It should only include numbers and operators (such as $+$, $\sqrt{}$).

(b) Do the same as part (a), except choose right endpoints as your sample points.

(c) Do the same as part (a), except choose midpoints as your sample points.

(d) Write the integral as a limit of Riemann sums using the definition of the integral. You can use whatever you like for sample points. You do not need to simplify your answer.

(3) Suppose the velocity of an object at time t is given by

$$v(t) = 27 - 3t^2$$

where t is in seconds and $v(t)$ is in feet/sec .

(a) Find the displacement of the object from $t = 0$ seconds to $t = 5$ seconds. Include units in your answer.

(b) Find the total distance the object travels from $t = 0$ seconds to $t = 5$ seconds,. Include units in your answer.

(4) Suppose that

$$\int_0^1 f(x)\, dx = 4$$

$$\int_1^2 f(x)\, dx = 5$$

$$\int_2^4 f(x)\, dx = 6$$

Use this to evaluate:

$$\int_0^2 x f\left(x^2\right)\, dx = ?$$

Justify your answer.

(5) Evaluate the following integrals. Show your work. Simplify your answers as appropritate. If you use a substition, write out the substition.

(a) $\int \frac{\sqrt{x}+x}{x^2}\, dx$

(b) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x\, dx$

(c) $\int_{-1}^{1} \frac{2}{1+x^2}\, dx$

(d) $\int \frac{\ln x}{x}\, dx$

(e) $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

(1) An oven is heating up. Suppose that

$$f(t) = \text{The temperature of the oven } (^\circ F) \text{ after } t \text{ minutes have passed}$$

and

$$g(t) = \text{The rate the temperature of the oven is increasing } (^\circ F/\text{min}) \text{ after } t \text{ minutes have passed}$$

Suppose that at time $t = 10$, the temperature of the oven is $90^\circ F$ (in other words, suppose that $f(10) = 90$).

(a) Using $f(t)$ and no other variables, write an expression representing the net increase in temperature of the oven (in $^\circ F$) from $t = 0$ minutes to $t = 30$ minutes.

$$f(30) - f(0) \qquad (^\circ F)$$

(b) Using $g(t)$ and no other variables, write an expression representing the net increase in temperature of the oven (in $^\circ F$) from $t = 0$ minutes to $t = 30$ minutes.

$$\int_0^{30} f(t)\,dt \qquad (^\circ F)$$

(c) Using $g(t)$ and no other variables, write an expression representing the temperature of the oven (in $^\circ F$) at $t = 30$ minutes.

$$f(10) + \int_{10}^{30} g(t)\,dt = 90 + \int_{10}^{30} g(t)\,dt \qquad (^\circ F)$$

(d) Using $g(t)$ and no other variables, write an expression representing the temperature of the oven (in $^\circ F$) at $t = 0$ minutes.

$$(^\circ F)$$

$$f(10) - \int_0^{10} g(t)\,dt = \boxed{90 - \int_0^{10} g(t)\,dt}$$

$$\text{or} \quad f(10) + \int_{10}^{0} g(t)\,dt = \boxed{90 + \int_{10}^{0} g(t)\,dt}$$

2 8 14 20 26

(2) Consider the definite integral

$$\int_2^{26} \sqrt{9+x^2}\,dx \qquad \Delta x = \frac{26-2}{4} = 6$$

(a) Write a Riemann sum approximating the above integral with 4 subdivisions, using left hand endpoints as sample points. You do NOT need to simplify or evaluate the sum, just write it out. You're final sum should not include any variables or letters or a summation sign. It should only include numbers and operators (such as $+$, $\sqrt{\ }$).

$$\left(\sqrt{9+2^2} + \sqrt{9+8^2} + \sqrt{9+14^2} + \sqrt{9+20^2}\right)6$$

(b) Do the same as part (a), except choose right endpoints as your sample points.

$$\left(\sqrt{9+8^2} + \sqrt{9+14^2} + \sqrt{9+20^2} + \sqrt{9+26^2}\right)6$$

(c) Do the same as part (a), except choose midpoints as your sample points.

$$\left(\sqrt{9+5^2} + \sqrt{9+11^2} + \sqrt{9+17^2} + \sqrt{9+23^2}\right)6$$

(d) Write the integral as a limit of Riemann sums using the definition of the integral. You can use whatever you like for sample points. You do not need to simplify your answer.

$$\int_2^{26} \sqrt{9+x^2}\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} \sqrt{9+\left(2+\frac{24k}{n}\right)^2} \cdot \frac{24}{n}$$

(3) Suppose the velocity of an object at time t is given by

$$v(t) = 27 - 3t^2$$

where t is in seconds and $v(t)$ is in feet/sec .

(a) Find the displacement of the object from $t = 0$ seconds to $t = 5$ seconds. Include units in your answer.

$$\int_0^5 v(t)\,dt = \int_0^5 (27-3t^2)\,dt = \left[27t - t^3\right]$$

$$= 135 - 125 = 10 \text{ feet}$$

(b) Find the total distance the object travels from $t = 0$ seconds to $t = 5$ seconds,. Include units in your answer.

$$|v(t)| = |27-3t^2| = \begin{cases} 27-3t^2, & t \le 3 \\ 3t^2-27, & t \ge 3 \end{cases}$$

$$\int_0^5 |v(t)|\,dt = \int_0^3 (27-3t^2)\,dt + \int_3^5 (3t^2-27)\,dt$$

$$= \left[27t - t^3\right]_0^3 + \left[t^3 - 27t\right]_3^5 = [81-27] + [98-54]$$

$$= 54 + 44 = 98 \text{ ft}$$

(4) Suppose that

$$\int_0^1 f(x)\,dx = 4$$

$$\int_1^2 f(x)\,dx = 5$$

$$\int_2^4 f(x)\,dx = 6$$

Use this to evaluate:

$$\int_0^2 x f(x^2)\,dx = ?$$

Justify your answer.

$$\int_0^2 x\, f(x^2)\,dx = \frac{1}{2}\int_0^4 f(u)\,du = \frac{1}{2}\int_0^4 f(x)\,dx$$

$u = x^2$

$du = 2x\,dx$

$x\,dx = \frac{1}{2}\,du$

x	$u = x^2$
0	0
2	4

$$= \frac{1}{2}\left(\int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx + \int_2^4 f(x)\,dx\right)$$

$$= \frac{1}{2}(4+5+6) = \frac{15}{2}$$

(5) Evaluate the following integrals. Show your work. Simplify your answers as appropritate. If you use a substition, write out the substition.

(a) $\int \frac{\sqrt{x}+x}{x^2}\, dx$

$$\int \frac{\sqrt{x}+x}{x^2}\,dx = \int x^{-3/2}\,dx + \int \frac{1}{x}\,dx = \boxed{-2x^{-\frac{1}{2}} + \ln x + C}$$

(b) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x\, dx$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x\,dx = \tan x \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \tan\frac{\pi}{4} - \tan\frac{-\pi}{4}$$
$$= 1 - (-1) = 2$$

(c) $\int_{-1}^{1} \frac{2}{1+x^2}\, dx$

$$\int_{-1}^{1} \frac{2}{1+x^2} = 2\arctan x \Big|_{-1}^{1} = 2\left(\frac{\pi}{4} - \frac{-\pi}{4}\right) = \pi$$

(d) $\int \frac{\ln x}{x}\, dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\int \frac{\ln x}{x}\,dx = \int u\,du = \frac{1}{2}u^2 + C$$
$$= \boxed{\frac{1}{2}(\ln x)^2 + C}$$

(e) $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$\frac{1}{\sqrt{x}}\,dx = 2\,du$

x	$u=\sqrt{x}$
1	1
4	2

$$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int_1^2 e^u\,du$$
$$= \boxed{2(e^2 - e)}$$