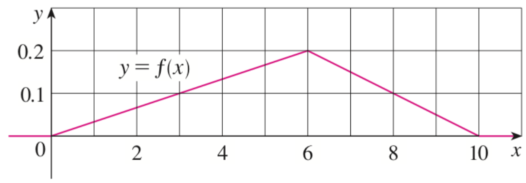
(1) (Problem 8 from the text) Suppose X is a random variable whose probability density function f is graphed below:



(a) State why f is a valid probability denstiy function.
(b) Find $P(X < 3)$
(c) Find $P(3 \leq X \leq 8)$
(d) Find the expected value of X.

(2) A random number generator generates a random number X between 3 and 7. Find (i) P($X \geq 5$), and (ii) the expected value of X given each of the following probability density functions for X:

(a) $f(x) = \begin{cases} \frac{1}{4} & 3 \leq x \leq 7 \\ 0 & \text{otherwise} \end{cases}$

(b) $f(x) = \begin{cases} \frac{x-3}{8} & 3 \leq x \leq 7 \\ 0 & \text{otherwise} \end{cases}$

(c) $f(x) = \begin{cases} \frac{7-x}{8} & 3 \leq x \leq 7 \\ 0 & \text{otherwise} \end{cases}$

(3) Suppose X is a random variable with probability distribution function

$$f(x) = \begin{cases} \frac{k}{x^3} & x \geq 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) What value must k have in order for this to be a valid probability distribution function? (Assume k has this value in the following parts of this problem).
(b) Find $P(4 \leq X \leq 5)$ (hint: you might find it easier to do part c before part b)
(c) Find a formula for the cumulative distribution function $F(x)$ of X, given by

$$F(x) = P(X \leq x)$$

(d) Find the expected value of X.
(e) The median value of a random variable X is the value m satisfying

$$P(X \leq m) = 0.5$$

Find the median value of X in this problem.

(4) Suppose a random variable X has the following probability distribution function:

$$f(x) = \begin{cases} \frac{k}{\sqrt{x}} & 0 < x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.
(a) What value must k have in order for this to be a valid probability distribution function? (Assume k has this value in the following parts of this problem). Notice that this problem involves a Type II improper integral.
(b) Find $P\left(X \leq \frac{1}{2}\right)$. (hint: you might find it easier to do part c before part b)
(c) Find a formula for the cumulative distribution function $F(x)$ for X, such that

$$F(x) = P(X \leq x)$$

(d) Find the expected value of X.
(e) The median value of a random variable X is the value m satisfying

$$P(X \leq m) = 0.5$$

Find the median value of X in this problem.

(5) Suppose that X represents the [redacted] time (in minutes) you must wait for a bus. Suppose the probability density function for X is given by

$$f(x) = \begin{cases} \frac{1}{10} e^{-x/10} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) What is the probability you will have to wait 10 minutes or less for a bus? (you might find it easier to do part (b) before part (a)

(b) Find a formula for the cumulative distribution function $F(x)$ of X, given by

$$F(x) = P(X \leq x)$$

(evaluate the appropriate improper integral, even if you already know what the answer is)

(c) What is the average (mean) time you have to wait for a bus?

(d) The median of a random variable X is defined to be the value m that satisfies

$$P(X \leq m) = 0.5$$

Find the median time you must wait for a bus.

(e) What is the probability that you will have to wait less than the median time for a bus?

(6) Let X be a random variable. Suppose that the cumulative distribution furnction for X is

$$P(X \leq x) = g(x) = \frac{1}{2} + \frac{1}{\pi} \arctan x$$

Notice this is the cumulate distribution function for X, not the probability density function.

(a) Find $P(X \leq 1)$

(b) Find $P(X \leq -1)$

(c) Find $P(-1 \leq X \leq 1)$

(d) Find the probability distribution for X.

(e) Find the expected value of X. Justify your answer fully, writing an improper integral.