(1) Do each of the following definite integrals two ways. First by computing the corresponding indefinite integral. Then again by doing the substitution in the definite integral directly and changing the limits of integration appropriately. The first one has been done as an example.

(a) $\int_0^{\sqrt{\pi}} x \sin x^2 \, dx$

Method #1

$$u = x^2,\ xdx = \frac{1}{2}du$$

$$\int x \sin x^2 \, dx = \frac{1}{2}\int \sin u du = -\frac{1}{2}\cos u + c = -\frac{1}{2}\cos x^2 + c$$

$$\int_0^{\sqrt{\pi}} x \sin x^2 \, dx = -\frac{1}{2}\left[\cos x^2\right]_0^{\sqrt{\pi}} = 1$$

Method #2

$$u = x^2,\ xdx = \frac{1}{2}du$$

$$\int_0^{\sqrt{\pi}} x \sin x^2 \, dx = \frac{1}{2}\int_0^{\pi} \sin du = -\frac{1}{2}\left[\cos u\right]_0^{\pi} = 1$$

(b) $\int_0^{\pi^2} \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}} \, dx$

(c) $\int_0^{\frac{\pi}{6}} \frac{\sin t}{\cos^2 t} \, dt$

(2) Suppose $\int_0^{10} f(x)\, dx = 12$. Find $\int_0^5 f(2x)\, dx$.

(3) Suppose $\int_0^8 f(t)\, dt = 20$. Find $\int_0^2 t^2 f\left(t^3\right) dt$.

(4) Suppose $\int_0^5 f(x)\, dx = 10$. Find $\int_0^{25} \frac{f\left(\sqrt{x}\right)}{\sqrt{x}} dx$

(5) Find $\int_0^{\pi} \cos x\, f(\sin x)\, dx$, where $f(x)$ is any integrable function. Justify your answer.

And do the following problems from section 5.5 of the text: 87-91, 94