Show appropriate work for all problems. If you use a substitution, show the substitution you use.

1. Evaluate the following integrals.

   (a) $\int_1^2 \frac{2x-x^2}{x^3}\,dx$

   (b) $\int (\sinh x - \sec x\,\tan x)\,dx$

   (c) $\int \frac{1+x}{1+x^2}\,dx$

2. Suppose

$$\int_1^2 f(x)\,dx = 3,\ \int_2^4 f(x)\,dx = 7,\ \text{and}\ \int_4^8 f(x)\,dx = -2$$

   Find

$$\int_1^4 \frac{f(\sqrt{x})}{\sqrt{x}}\,dx$$

3. Suppose we are growing a population of yeast, and that

$$f(t) = \text{The rate at which the population is growing (in grams/min) at time } t \text{ (in minutes)}$$

   (a) What does the following mean? Give a complete sentence, and include appropriate units.

$$\int_0^{20} f(t)\,dt = 17$$

   (b) Suppose the statement in (a) is true, and that after 20 minutes the population of yeast is 50 grams. What was the original population at $t = 0$ minutes?

Show appropriate work for all problems. If you use a substitution, show the substitution you use.

1. Evaluate the following integrals.

(a) $\int_1^2 \frac{2x-x^2}{x^3}\,dx$

$\frac{2x-x^2}{x^3} = \frac{2x}{x^3} - \frac{x^2}{x^3} = 2x^{-2} - \frac{1}{x}$

$\int_1^2 \frac{2x-x^2}{x^3}\,dx = \int_1^2 \left(2x^{-2} - \frac{1}{x}\right)dx = \left[-2x^{-1} - \ln|x|\right]_1^2 = (-1-\ln 2) - (-2-0) = 1 - \ln 2$

(b) $\int (\sinh x - \sec x \tan x)\,dx$

$= \boxed{\cosh x - \sec x + C}$

(c) $\int \frac{1+x}{1+x^2}\,dx$

$= \int \frac{1}{1+x^2}dx + \int \frac{x}{1+x^2}dx$

$u = 1+x^2 \quad du = 2x\,dx$

$\to \int \frac{x}{1+x^2}dx = \frac{1}{2}\int \frac{2x}{1+x^2}dx = \frac{1}{2}\int \frac{1}{u}du$

$= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln(1+x^2) + C$

$\int \frac{1+x}{1+x^2}dx = \boxed{\arctan x + \frac{1}{2}\ln(1+x^2) + C}$

2. Suppose

$$\int_1^2 f(x)\,dx = 3, \quad \int_2^4 f(x)\,dx = 7, \text{ and } \int_4^8 f(x)\,dx = -2$$

Find

$$\int_1^4 \frac{f(\sqrt{x})}{\sqrt{x}}\,dx$$

Let $u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}dx$

$\frac{1}{\sqrt{x}}dx = 2\,du$

$\int_1^4 \frac{f(\sqrt{x})}{\sqrt{x}}dx = 2\int_1^2 f(u)\,du = 2(3) = 6$

Don't forget to change the limits

3. Suppose we are growing a population of yeast, and that

$f(t)$ = The rate at which the population is growing (in grams/min) at time t (in minutes)

(a) What does the following mean? Give a complete sentence, and include appropriate units.

$$\int_0^{20} f(t)\,dt = 17$$

From 0 to 20 minutes, the population of yeast grew 17 grams bigger

(b) Suppose the statement in (a) is true, and that after 20 minutes the population of yeast is 50 grams. What was the original population at $t = 0$ minutes?

$50 - 17 =$ 33 grams