

Calculus Integration Worksheet

Part A: Basic Integration

1. Find the integral of the following functions:

- a) $\int (3x^2 - 2x + 4) \, dx$ $\int (3x^2 - 2x + 4) \, dx$
- b) $\int (1x+5) \, dx$ $\int \left(\frac{1}{x} + 5\right) \, dx$ $\int (x+5) \, dx$
- c) $\int \sin(x) \, dx$ $\int \sin(x) \, dx$
- d) $\int e^{2x} \, dx$ $\int e^{2x} \, dx$

Part B: Integration by Substitution

2. Evaluate the following integrals using substitution:

- a) $\int x \cos(x^2) \, dx$ $\int x \cos(x^2) \, dx$
- b) $\int 3x^2 + 1 \, dx$ $\int \frac{3x}{x^2 + 1} \, dx$ $\int x^2 + 13x \, dx$
- c) $\int e^{3x+1} \, dx$ $\int e^{3x+1} \, dx$
- d) $\int \frac{1}{\sqrt{4-x^2}} \, dx$ $\int 4-x^2 \, dx$

Part C: Integration by Parts

3. Solve the following integrals using integration by parts:

- a) $\int x e^x \, dx$ $\int x e^x \, dx$
- b) $\int \ln(x) \, dx$ $\int \ln(x) \, dx$
- c) $\int x \sin(x) \, dx$ $\int x \sin(x) \, dx$
- d) $\int x^2 \cos(x) \, dx$ $\int x^2 \cos(x) \, dx$

Part D: Definite Integrals

4. Evaluate the following definite integrals:

- a) $\int_0^2 (x^3 - 4x) \, dx$ $\int_0^2 (x^3 - 4x) \, dx$
- b) $\int_1^3 \frac{1}{x^2} \, dx$ $\int_1^3 \frac{1}{x^2} \, dx$
- c) $\int_0^{\pi} \sin(x) \, dx$ $\int_0^{\pi} \sin(x) \, dx$
- d) $\int_{-1}^1 e^{-x^2} \, dx$ $\int_{-1}^1 e^{-x^2} \, dx$

Part E: Applications

5. Apply integration to solve the following problems:

- a) Find the area under the curve $y = x^2 - 2x + 1$ from $x = 0$ to $x = 3$.
- b) Determine the volume of the solid formed by rotating the region bounded by $y = x^2$ and the x-axis from $x = 0$ to $x = 2$ about the x-axis.

- c) Compute the length of the curve $y = \ln(x)$ from $x=1$ to $x=e$.
- d) Find the work done by a force $F(x) = 4x - x^2$ as it moves an object from $x=0$ to $x=3$.