**Workbook: Combining Functions – A Simple Guide to Composite Functions**

**Questions**

**Section 1: Introduction to Composite Functions**

* **What Are Composite Functions?**
  + Explanation of composite functions.
  + Notation and basic understanding.
  + Visual representation (e.g., diagrams showing how one function’s output feeds into another).
* **Example Problems**
  + Basic problems to practice the concept.
  + Fill-in-the-blank exercises to reinforce notation and understanding.

**Section 2: Step-by-Step Examples**

* **Example 1: Basic Composite Function**
  + Step-by-step guide on solving a basic composite function.
  + Practice problems with space for students to work out answers.
  + Answer key with detailed explanations.
* **Example 2: Real-Life Application**
  + A scenario-based example showing the application of composite functions.
  + Exercises related to real-life applications (e.g., economics, physics).
  + Reflection questions to help students connect the concept to other subjects.

**Section 3: Advanced Concepts**

* **Decomposing Functions**
  + Exercises on breaking down composite functions into their component functions.
  + Problems to identify f(x)f(x)f(x) and g(x)g(x)g(x) from a given composite function.
* **Inverses of Composite Functions**
  + Introduction to finding inverses.
  + Practice problems with answers and explanations.

**Section 4: Real-Life Scenarios**

* **Scenario 1: Economics**
  + A detailed problem involving composite functions in an economic context.
  + Problem-solving space and guided steps.
* **Scenario 2: Physics**
  + A physics-based problem using composite functions.
  + Diagrams and charts to visualize the problem.
* **Scenario 3: Programming**
  + An example of function composition in programming.
  + Coding exercises (pseudo-code or real programming language examples).

**Section 5: Interactive Exercises**

* **Function Composition Puzzles**
  + Engaging puzzles to solve using composite functions.
  + Mix-and-match exercises (match the function to its composite).
* **Multiple-Choice Questions**
  + A series of multiple-choice questions to test understanding.
  + Detailed explanations for each answer.

**Section 6: Reflection and Application**

* **Reflection Questions**
  + Questions to encourage students to think about how they might use composite functions in different areas.
* **Student Submissions**
  + Space for students to create their own composite function problems.
  + A section for peer review or teacher feedback.

**Section 7: Summary and Cheat Sheet**

* **Summary**
  + Recap of key concepts.
  + Important tips and tricks for solving composite functions.
* **Downloadable Cheat Sheet**
  + A condensed version of the workbook’s main points.
  + Printable version for quick reference.

**Section 8: Assessment**

* **Final Quiz**
  + A quiz covering all sections of the workbook.
  + Graded answers with explanations.

**Design Elements:**

* Include graphs, charts, and visuals where applicable.
* Use plenty of space for students to work out problems.
* Incorporate QR codes linking to interactive online content (like videos or further reading).

**Interactive Elements:**

* Use Google Forms or other online tools for interactive quizzes.
* Provide downloadable worksheets for offline work.

**Format:**

* PDF version for easy printing.
* Editable version (e.g., Word or Google Docs) for customization.

**Solutions**

**Workbook Answers: Combining Functions – A Simple Guide to Composite Functions**

**Section 1: Introduction to Composite Functions**

* **Example Problems:**
  + **Problem 1:** If f(x)=3x+2f(x) = 3x + 2f(x)=3x+2 and g(x)=x2g(x) = x^2g(x)=x2, find (f∘g)(x)(f \circ g)(x)(f∘g)(x).
    - **Answer:** (f∘g)(x)=f(g(x))=f(x2)=3x2+2(f \circ g)(x) = f(g(x)) = f(x^2) = 3x^2 + 2(f∘g)(x)=f(g(x))=f(x2)=3x2+2.
    - **Explanation:** Substitute g(x)=x2g(x) = x^2g(x)=x2 into f(x)=3x+2f(x) = 3x + 2f(x)=3x+2.
  + **Problem 2:** If f(x)=xf(x) = \sqrt{x}f(x)=x​ and g(x)=x+1g(x) = x + 1g(x)=x+1, find (g∘f)(x)(g \circ f)(x)(g∘f)(x).
    - **Answer:** (g∘f)(x)=g(f(x))=g(x)=x+1(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{x} + 1(g∘f)(x)=g(f(x))=g(x​)=x​+1.
    - **Explanation:** Substitute f(x)=xf(x) = \sqrt{x}f(x)=x​ into g(x)=x+1g(x) = x + 1g(x)=x+1.

**Section 2: Step-by-Step Examples**

* **Example 1: Basic Composite Function**
  + **Problem:** Find (f∘g)(x)(f \circ g)(x)(f∘g)(x) where f(x)=2x+3f(x) = 2x + 3f(x)=2x+3 and g(x)=x2g(x) = x^2g(x)=x2.
    - **Answer:** (f∘g)(x)=f(g(x))=2x2+3(f \circ g)(x) = f(g(x)) = 2x^2 + 3(f∘g)(x)=f(g(x))=2x2+3.
    - **Explanation:** Substitute g(x)=x2g(x) = x^2g(x)=x2 into f(x)=2x+3f(x) = 2x + 3f(x)=2x+3.
  + **Problem:** Find (g∘f)(x)(g \circ f)(x)(g∘f)(x).
    - **Answer:** (g∘f)(x)=g(f(x))=(2x+3)2=4x2+12x+9(g \circ f)(x) = g(f(x)) = (2x + 3)^2 = 4x^2 + 12x + 9(g∘f)(x)=g(f(x))=(2x+3)2=4x2+12x+9.
    - **Explanation:** Substitute f(x)=2x+3f(x) = 2x + 3f(x)=2x+3 into g(x)=x2g(x) = x^2g(x)=x2.
* **Example 2: Real-Life Application**
  + **Problem:** Given g(x)=2xg(x) = 2xg(x)=2x (hours studied) and f(x)=5x+20f(x) = 5x + 20f(x)=5x+20 (test score based on study time), find (f∘g)(x)(f \circ g)(x)(f∘g)(x).
    - **Answer:** (f∘g)(x)=10x+20(f \circ g)(x) = 10x + 20(f∘g)(x)=10x+20.
    - **Explanation:** Substitute g(x)=2xg(x) = 2xg(x)=2x into f(x)=5x+20f(x) = 5x + 20f(x)=5x+20.

**Section 3: Advanced Concepts**

* **Decomposing Functions**
  + **Problem:** Given h(x)=4x2+8x+5h(x) = 4x^2 + 8x + 5h(x)=4x2+8x+5, decompose it into two functions f(x)f(x)f(x) and g(x)g(x)g(x) such that h(x)=(f∘g)(x)h(x) = (f \circ g)(x)h(x)=(f∘g)(x).
    - **Answer:** f(x)=4x+5f(x) = 4x + 5f(x)=4x+5, g(x)=x2+2xg(x) = x^2 + 2xg(x)=x2+2x.
    - **Explanation:** Identify that g(x)=x2+2xg(x) = x^2 + 2xg(x)=x2+2x and substitute into f(x)f(x)f(x) to get h(x)h(x)h(x).
* **Inverses of Composite Functions**
  + **Problem:** If f(x)=3x−4f(x) = 3x - 4f(x)=3x−4 and g(x)=x+2g(x) = x + 2g(x)=x+2, find (f∘g)−1(x)(f \circ g)^{-1}(x)(f∘g)−1(x).
    - **Answer:** (f∘g)−1(x)=x+43−2(f \circ g)^{-1}(x) = \frac{x + 4}{3} - 2(f∘g)−1(x)=3x+4​−2.
    - **Explanation:** First find (f∘g)(x)=3(x+2)−4=3x+2(f \circ g)(x) = 3(x + 2) - 4 = 3x + 2(f∘g)(x)=3(x+2)−4=3x+2. Then, solve y=3x+2y = 3x + 2y=3x+2 for xxx to get the inverse.

**Section 4: Real-Life Scenarios**

* **Scenario 1: Economics**
  + **Problem:** If the demand function is D(p)=100−2pD(p) = 100 - 2pD(p)=100−2p and the price function is p(x)=5x+10p(x) = 5x + 10p(x)=5x+10, find the composite demand function D(p(x))D(p(x))D(p(x)).
    - **Answer:** D(p(x))=100−2(5x+10)=100−10x−20=80−10xD(p(x)) = 100 - 2(5x + 10) = 100 - 10x - 20 = 80 - 10xD(p(x))=100−2(5x+10)=100−10x−20=80−10x.
    - **Explanation:** Substitute p(x)p(x)p(x) into D(p)D(p)D(p) to get the composite function.
* **Scenario 2: Physics**
  + **Problem:** If velocity v(t)=10t+5v(t) = 10t + 5v(t)=10t+5 (where ttt is time) and time as a function of distance is t(s)=s5t(s) = \frac{s}{5}t(s)=5s​, find v(t(s))v(t(s))v(t(s)).
    - **Answer:** v(t(s))=10(s5)+5=2s+5v(t(s)) = 10\left(\frac{s}{5}\right) + 5 = 2s + 5v(t(s))=10(5s​)+5=2s+5.
    - **Explanation:** Substitute t(s)=s5t(s) = \frac{s}{5}t(s)=5s​ into v(t)v(t)v(t).
* **Scenario 3: Programming**
  + **Problem:** Write a composite function in pseudo-code where f(x)=x2f(x) = x^2f(x)=x2 and g(x)=x+3g(x) = x + 3g(x)=x+3.
    - **Answer:**

pseudo

Copy code

function composite(x):

return (x + 3) ^ 2

* + - **Explanation:** The pseudo-code applies g(x)g(x)g(x) first, then squares the result to simulate f(g(x))f(g(x))f(g(x)).

**Section 5: Interactive Exercises**

* **Function Composition Puzzles**
  + **Puzzle 1:** Match f(x)=2x+1f(x) = 2x + 1f(x)=2x+1 with g(x)=x2g(x) = x^2g(x)=x2. Find (f∘g)(x)(f \circ g)(x)(f∘g)(x).
    - **Answer:** (f∘g)(x)=2x2+1(f \circ g)(x) = 2x^2 + 1(f∘g)(x)=2x2+1.
  + **Puzzle 2:** Match f(x)=xf(x) = \sqrt{x}f(x)=x​ with g(x)=x+4g(x) = x + 4g(x)=x+4. Find (g∘f)(x)(g \circ f)(x)(g∘f)(x).
    - **Answer:** (g∘f)(x)=x+4(g \circ f)(x) = \sqrt{x} + 4(g∘f)(x)=x​+4.
* **Multiple-Choice Questions**
  + **Question 1:** What is (f∘g)(x)(f \circ g)(x)(f∘g)(x) if f(x)=2x+3f(x) = 2x + 3f(x)=2x+3 and g(x)=x2g(x) = x^2g(x)=x2?
    - **Answer:** (B) 2x2+32x^2 + 32x2+3.
  + **Question 2:** If f(x)=3xf(x) = 3xf(x)=3x and g(x)=x−1g(x) = x - 1g(x)=x−1, what is (g∘f)(x)(g \circ f)(x)(g∘f)(x)?
    - **Answer:** (A) 3x−13x - 13x−1.

**Section 6: Reflection and Application**

* **Reflection Questions**
  + **Question:** How might you use composite functions in understanding compound interest in finance?
    - **Answer:** Composite functions can be used to model the interest earned over time by applying the interest formula multiple times, considering how the output of one period becomes the input for the next.
* **Student Submissions**
  + **Example:** If a student submits f(x)=2x+5f(x) = 2x + 5f(x)=2x+5 and g(x)=x2g(x) = \frac{x}{2}g(x)=2x​, the composite function (f∘g)(x)(f \circ g)(x)(f∘g)(x) would be x+5x + 5x+5.

**Section 7: Summary and Cheat Sheet**

* **Summary**
  + Review the key points such as the process of function composition, common mistakes to avoid, and tips for solving complex problems.
* **Downloadable Cheat Sheet**
  + **Answer Key:** A summary of important formulas and steps for solving composite functions.

**Section 8: Assessment**

* **Final Quiz**
  + **Question 1:** Given f(x)=x2+1f(x) = x^2 + 1f(x)=x2+1 and g(x)=3x−2g(x) = 3x - 2g(x)=3x−2, find (f∘g)(x)(f \circ g)(x)(f∘g)(x).
    - **Answer:** 9x2−12x+59x^2 - 12x + 59x2−12x+5.
  + **Question 2:** If f(x)=xf(x) = \sqrt{x}f(x)=x​ and g(x)=4x+7g(x) = 4x + 7g(x)=4x+7, what is (g∘f)(x)(g \circ f)(x)(g∘f)(x)?
    - **Answer:** 4x+74\sqrt{x} + 74x​+7.